Proceedings of the I·R·E

OCTOBER, 1944

VOLUME 32 NUMBER 10

SEVENTH CANADIAN VICTORY LOAN

INVEST IN VICTORY

OTICE TO VOTING MEMBERS: Those who have not already sent in their ballots on the constitutional amendments are urged to do so immediately.

Institute of Radio Engineers

Expressions of the viewpoints of leaders and pioneers in the radio-and-electronic industry will naturally be of interest and assistance to the active engineers in that field. Accordingly there is here presented in the form in which it was received from the President of The Blue Network Company an analysis of the role of the radio engineers in broadcasting.

The Editor

Broadcast-Engineering Prospects

MARK WOODS

Although the technical aspects of the engineer's work in many cases are as confusing to me as they are to the average layman (I get a clear picture of millivolts and watts only when I see how much money is involved), as president of one one of the major radio networks I am more keenly aware than most people of the basic changes in our civilization for which engineers are responsible.

If I consider the entertainment field alone, I am amazed at the effect of technical developments on our fundamental social habits. Few people realize that from the dawn of civilization there was no major change in entertainment until approximately 40 years ago when the engineer developed the talking machine, and presented man with the first medium of mechanical entertainment in the home. For the first time, the talent of professional artists was made available outside the theatre and the concert hall.

The small cities and villages were the chief beneficiaries of the next development, the silent motion picture, which gave their inhabitants the opportunity to see at frequent intervals the actors and actresses who seldom, if ever, graced the local Town Hall. An artist might appear in person only once a year but the nickleodeon presented him in "reasonable facsimile" at much more frequent intervals.

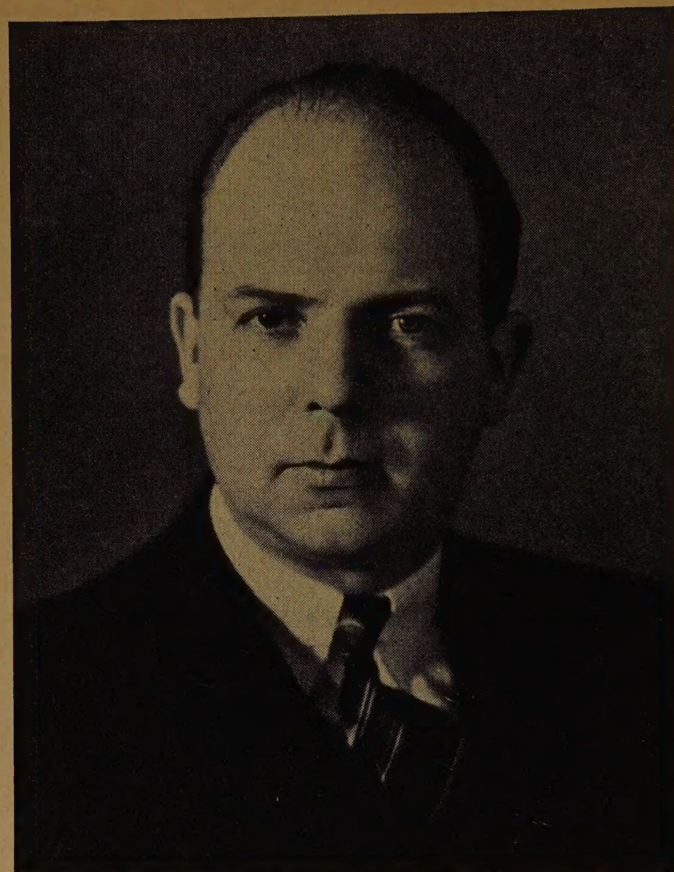
Going back to the home again, radio wrought a really revolutionary change in entertainment habits in that it brought the actual voice of the artist within hearing distance of our armchairs. Drama, discussion, last-minute news, and the give and take of the audience participation show added zest to the entertainment fare in the home, formerly consisting of musical recordings. The "fluffs" of the radio actor, the stuttering of the amateur in the audience, the obvious nervousness of the announcer with an extra moment to fill—all these, instead of evoking comparisons with the smoothness of recorded performances, merely emphasized the "live" quality of the entertainment.

The engineer again changed the pattern of mass entertainment when he combined both sight and sound in the talking motion picture.

Now, immediately on the horizon is television with the added asset of a medium which not only combines sight and sound but can be offered in the home at the same time the televised event is taking place. One of the infinite possibilities of television which the layman sometimes forgets is its use as a medium of education. As the silent motion picture brought the artist to the smallest village in the country, so television can take the outstanding educators of our time, completely equipped with the charts and pictures that illustrate their points, to the smallest school.

Starting with the psychological premise that we are all born lazy, it is not inconceivable that television will bring about radical changes in our social habits. Since it is easier to stay home than to go out, the young people of the next 15 years, discovering that television offers them the same quality of entertainment that they can find on the outside, might very well lose the mob instinct of their elders, the urge to mingle with large groups of people.

We may expect to see these changes in the near future because of the technical advances which are always made in wartime when economy is ruled out and money is spent freely on research. When the many accomplishments of the engineer, now veiled in secrecy, are made available to civilians, people may not be aware of their importance but they will inevitably feel their effect in every phase of their lives.



William Litell Everitt

Board of Directors—1944

William Litell Everitt was born in Baltimore, Maryland, on April 14, 1900. He served in World War I in the United States Marine Corps and was graduated in electrical engineering from Cornell University in 1922, serving as an instructor in that subject from 1920 to 1922. He was an engineer with the North Electric Manufacturing Company of Galion, Ohio, from 1922 to 1924, where he was in charge of the design and development of their relay automatic public switchboard exchanges. In 1924, he became an instructor in electrical engineering at the University of Michigan, remaining there until 1926 when he went to the Ohio State University to take charge of their instruction in communication engineering.

He holds the degrees of M.S. from the University of Michigan and Ph.D. from the Ohio State University. He served with the department of development and research of the American Telephone and Telegraph Company during the summers from 1925 to 1930. He has been a consultant for a number of broadcast stations and radio manufacturing companies.

He is the author of "Communication Engineering," and the section on "Wire Telephony and Telegraphy" of the "Standard Handbook for Electrical Engineers." He is the editor of a text on "Fundamentals of Radio."

Dr. Everitt originated and directed at Ohio State the annual Broadcast Engineering Conference which became the focus of engineering discussion in this field and with which the Institute co-operated.

In 1940, he became a member of the Communications Section of the National Defense Research Committee.

Since 1942, Dr. Everitt has been on leave from Ohio State as Director of Operational Research with the Signal Corps of the United States Army.

A director of the Institute since 1942, he is also a member of the Board of Editors and chairman of the Education and Awards committees.

He is a Fellow of the American Institute of Engineers and was chairman of their Communication Committee from 1937–1939. He was a member of the National Council of Tau Beta Pi and is also a member of Sigma Xi and Eta Kappa Nu.

Dr. Everitt became an Associate member of the Institute of Radio Engineers in 1925, a Member in 1929, and a Fellow in 1938.

Engineering Education*

Suggested Topics for Discussion

Foreword

In a recent article[1] it was proposed that the individual sections discuss the design of engineering curricula with the same basic approach they would apply to the design of other engineering products. A number of the sections have agreed to devote a meeting to this subject.

The Education Committee hopes these discussions will consider the fundamental problems to the fullest extent possible. It should be remembered that the primary purpose of the college is not to disseminate information but to *train minds*. Information may become obsolete but the ability to analyze and synthesize is never lost.

To stimulate discussion, the Committee proposes here certain fundamental questions. Partial answers are given where they are necessary to develop subsequent questions, or suggest a line of thought. These partial answers should be considered as purely tentative. As new answers are developed, they will suggest additional questions.

The first four questions are raised to give direction to the discussion. At the end, questions on certain specific courses such as mathematics and electronics are suggested as examples, since actual planning of curricula must finally end in determination of the details of specific cases.

Before proceeding with a discussion of the design of engineering curricula, question 1 should be answered:

Question 1. What are the successive steps in the design and production of an engineering product?

Tentative Answer:

1. A need presents itself and is recognized.
2. A decision is made on the general practical characteristics of the product.
3. The designer surveys the situation, determines what has been done before, what needs have not been met, and what changes should be made from previous practice.
4. One or more plans are outlined to accomplish the desired results.
5. The materials and labor required for each plan are ascertained and the plans compared on the basis of the relative cost and the results to be expected.
6. Engineering judgment is applied to determine which one of the plans is most feasible and whether any of them will produce sufficient improvement over old products to expect public acceptance.
7. The management of a manufacturing concern must be persuaded to produce the proposed product.
8. The design is put into production.
9. The product is marketed.
10. Experience in public use is reflected back into plans for periodic improvement.

Question 2. Which of these successive steps should be considered in the design of engineering curricula?

Question 3. Assuming there is a need for engineering education, which of the steps in its design are best suited for an evening's discussion?

Tentative Answer:

1. Decision on attributes desired in the product (the engineering graduate).
2. Determination of needs which should be met by a curriculum.
3. Discussion of plans for meeting needs.
4. Discussion of problems of cost of proposed plan in student time, teaching requirements, and laboratory facilities.
5. Discussion of how to put the plan into production.

Question 4. What should be the characteristics of an engineering graduate on leaving college? (Do not expect the impossible, remember he is still young and his education is not completed.)

Tentative Answer:

1. A love for the profession.
2. Moral integrity.
3. Breadth of background.
   a. Technical knowledge.
   b. Knowledge of human relations.
4. Ability and desire to acquire knowledge and improve himself. Possible sources of knowledge.
   a. People
   b. Books
   c. Apparatus
   d. Original thinking
5. Common sense or judgment.
6. Originality.
7. Clarity of expression, the ability to present ideas persuasively.
8. An engaging personality, the ability to make friends, inspire confidence, and work in a team.
9. The ability to enjoy life.

Question 5. How can a love for and loyalty to the engineering profession be developed?

Partial Answer:

1. Instruction in the history, objectives, and accomplishments of engineering.

* Decimal classification: Original manuscript received by the Institute, September 5, 1944. This report was prepared by the Education Committee of the I.R.E.

[1] W. L. Everitt, "The Phoenix—A challenge to engineering education," Proc. I.R.E., vol. 32, pp. 509–513; September, 1944.

2. Lectures by, and interviews with, practicing engineers.
3. Better instruction in what engineering is and what an engineer does.
4. Instruction in the responsibilities of the engineer.

Question 6. What can the college do to develop moral integrity?

Answer should be determined by discussion. This may include the responsibilities of an engineer, the influence of an honor system, the need for Institute formulation of codes of ethics, faculty-student relations, etc.

Question 7. What fields of technical knowledge are necessary for the electrical engineer who expects to practice in the fields of communications and electronics?

Partial Answer:

1. The nature of the physical world (physics and chemistry).
2. The quantitative description of physical phenomena and the methods to be used in prediction of results from causes (mathematics).
3. Materials and their processing.
4. Specific electrical subject matter.
   a. The principles of conduction currents (circuits).
   b. The principles of convection currents (electronics).
   c. The principles of displacement currents (electromagnetic fields).
   d. The principles of energy conversion (coupling between electrical, mechanical and other systems).
   e. Practical applications combining a, b, c, and d.
   f. Experimental techniques.
5. Engineering economics (the computation of costs).
6. Sources of information.

Question 8. What fields of knowledge of human relations are necessary for engineers?

Partial Answer:

1. History
2. Literature
3. Psychology
4. Economics
5. Government
6. Geography
7. Law
8. Labor relations
9. Administration

Question 9. How can so many fields of human knowledge, usually taught in separate departments, be introduced in an engineering education so their interrelationship is appreciated? Should the essentials of several coordinated fields be combined in one or more courses? Should this instruction precede or follow technical instruction or should it run concurrently throughout the whole curricula as an integrating influence?

Question 10. What should be the relative distribution of time between technical and humanistic studies?

Question 11. How should courses be taught to fit the future engineer for actual applications and so the student is stimulated to continue to acquire knowledge after his college career is ended?

Question 12. How can a student, with his limited background, be taught engineering judgment?

Question 13. How can personality be developed in college, and how can positive or negative progress be indicated to the student? What attributes of personality should be stressed, such as consideration for others (sometimes called etiquette), tact, likeability, teamwork, dependability, punctuality, poise, get-along ability?

Question 14. Is the heavy work load, common to most engineering curricula, conducive to the well-rounded development of personality or should the load be reduced?

Question 15. How can clarity of expression be developed?

Tentative Answer:

By training in
1. English composition
2. Pictorial expression
   a. Free-hand
   b. Mechanical
3. Report Writing
4. Oral Expression
5. Dictation

Question 16. How can originality or the ability to perform engineering synthesis be developed?

Partial Answer:

1. By courses in design.
2. By senior projects or theses.
3. By increased latitude in laboratory work.
4. By temporary assignments as laboratory foreman or assistant instructor.
5. By co-operative courses or summer work in industry.

Question 17. How can the engineer be taught the way to enjoy life as an educated man and a member of his community?

This should be discussed thoroughly. Little has been done formally in the colleges. The influence of extracurricular activities, electives in the arts, the development of hobbies, and the ability to mix well in society are possible approaches.

Question 18. What alternative programs should be provided for engineers? Should the program for men expecting to enter research and development differ from that for men expecting to enter production, operation, and sales? If such a differentiation is made, at what point in the curricula should it be introduced? Should graduate work be normally expected for research and development?

Question 19. What are the primary purposes of laboratory work, and how should it be conducted?

Question 20. What is the ideal preparation for, and characteristics of, an engineering teacher? In your experience, did your teachers meet these requirements?

Question 21. What are the practical limits in time for an engineering education?

Question 22. What topics in mathematics should be taught? How should its relation to the analysis of physical systems (the end result desired by engineers) be developed?

Question 23. What are the important topics in electronics and electric fields, not taught previous to the war, which should be added to the curricula, assuming

1. a four-year course
2. a graduate year beyond the bachelor's degree.

Question 24. What should be the contents of a course such as the "Philosophy and Methods of Engineering" to acquaint students in other fields with engineering?

Question 25. What needs in radio education will not be met by redesign of college curricula and how may they be met? (For example, technician training and instruction of practicing engineers.) This may develop material for discussions at subsequent meetings.

Question 26. Should further meetings be devoted to discussions of educational problems?

Combination of Amplitude and Frequency Modulation for Communication in Seismograph Exploration for Petroleum Reservoirs*

EARLEY M. SHOOK†*, ASSOCIATE, I.R.E., ROBERT W. OLSON†**, NONMEMBER, I.R.E., AND ROBERT B. KERR†***, NONMEMBER, I.R.E.

Summary—The reflection seismograph method of exploring and contouring various subsurface geological beds suitable for petroleum deposits is reviewed briefly. Instrumentation techniques are passed over briefly with the exception of equipment as described which is devised to transmit and receive by frequency modulation certain electrical impulses generated coincident with the detonation of the dynamite charge and with the arrival of the seismic impulse at the earth's surface immediately above the explosive charge. Amplitude modulation of voice signals was devised for transmission and reception over the same radio channel. The combination of the two schemes of modulation provided suitable voice reception without interfering with the frequency-modulation system to record the time break and uphole geophone electrical impulses free of static and accurate to 1/1000 of a second. The apparatus provides also for transmission of these impulses and voice signals by wire transmission with the same magnitude, clarity, and precision by means of simple switching arrangements. The 10-watt input amplitude-modulation—frequency-modulation transmitter devised is ample for the purpose up to 1 mile which is sufficient for reflection seismograph exploration.

I. Introduction

GEOPHYSICAL exploration for locating subsurface petroleum reservoirs by the seismic method presents many problems, some of which can be solved satisfactorily by techniques familiar to the radio and electronic scientists. The specific problems of the communication of speech and certain generated electrical impulses in the presence of severe static and sound interferences may prove of sufficient interest to the radio engineers as to merit a discussion of them in the PROCEEDINGS.

The specific problems to be discussed in this paper arose in connection with seismic exploration for the delineation of subsurface geological structures favorable to the accumulation of oil and gas deposits. This method of exploration requires, among others, a shooting and recording truck, generally spaced apart from 1000 to 3000 feet. The former is equipped with various paraphernalia for loading and detonating the dynamite in addition to certain parts of the communication apparatus. The recording truck contains the multiple-channel seismic amplifiers, oscillograph, photographic and the remaining parts of the communication equipment. A number of seismic geophones or detectors for the most part are buried at the surface of the ground and appropriately spaced. There are usually from 8 to 24 multiple channels, and one or more detectors are connected with cables to each channel. This array of detectors is known as the "spread" and usually is placed linearly between the recording and shooting trucks.

Speech communication between members of the recording and shooting trucks is necessary in order that the different functions of the two groups may be co-ordinated and synchronized. Also, it is necessary to record photographically the exact instant of time at which the dynamite is detonated. An electrical impulse, known as the "time break," is produced at this instant of detonation.

It is generally customary to place a seismic detector at the surface of the ground adjacent to the hole in which is placed the charge of dynamite. This detector

* Decimal classification: R414×R536. Original manuscript received by the Institute, March 14, 1944; revised manuscript received, June 26, 1944.

† All on leave from the Geophysical Laboratory, Magnolia Petroleum Company, Dallas, Texas; * Lieutenant Commander, United States Navy, ** Naval Ordnance Laboratory, *** Lieutenant (jg), United States Navy. In the absence of these men, the present article was prepared from their work by Dr. J. P. Minton, associate director, field research department, Magnolia Petroleum Company.

serves to permit the photographic recordation of the arrival of those seismic waves generated by the explosion and traveling vertically from the shot location to the surface. It is desired to determine the very first instant of this event at the uphole geophone. The time required for these particular waves to reach the surface is generally referred to as the "uphole time" and the electrical impulse generated by the detector because of excitation by these waves is called the uphole detector impulse.

Both the time-break and uphole-detector impulses are transmitted over the speech-transmission channel to the recording truck for photographic recording on the seismogram.

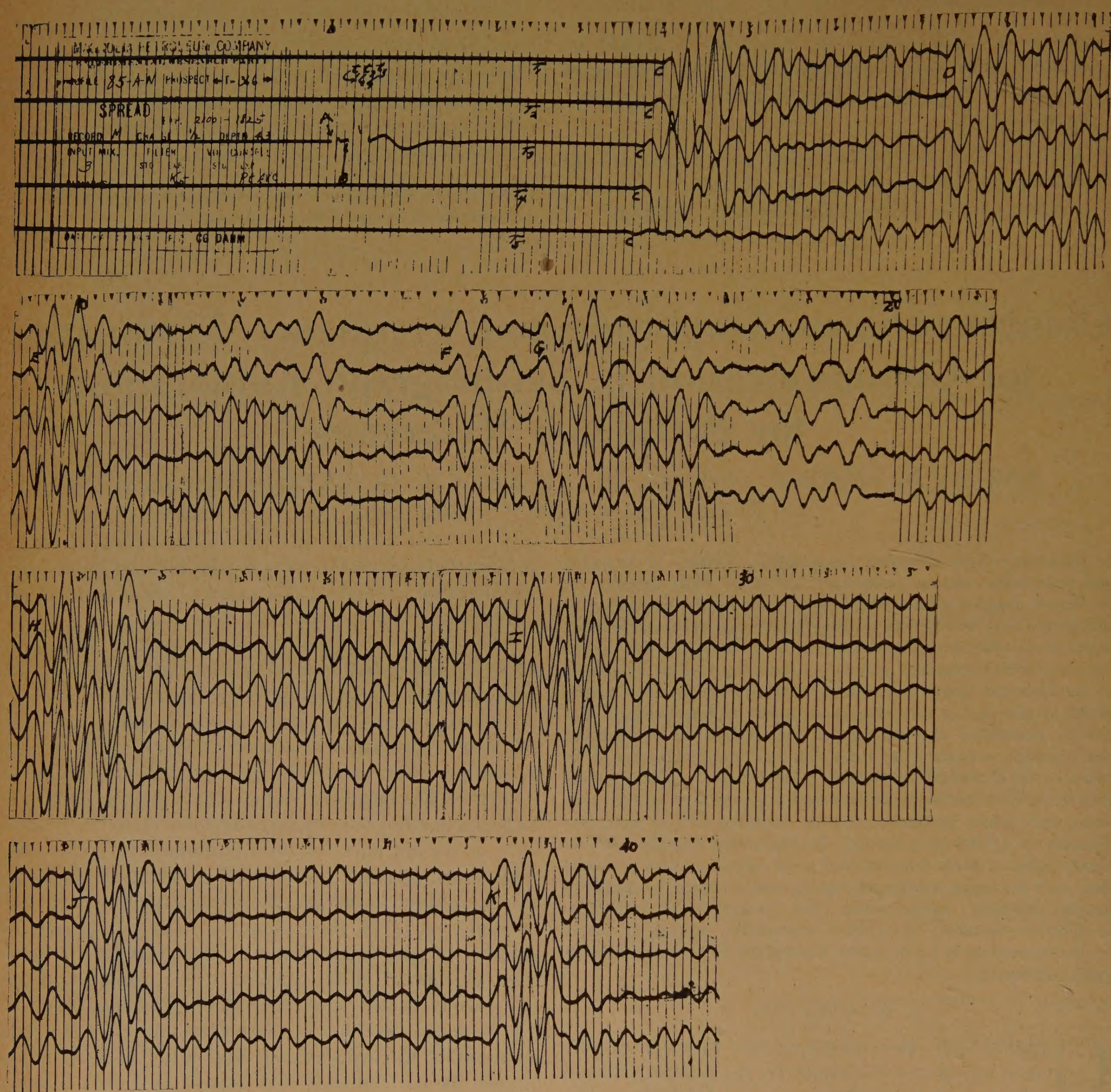


Fig. 1.—Multiple trace seismogram.
A—Time break.
B—Uphole geophone break.
C—Arrival of refraction wave at various geophones by shortest time paths.
D–K—Arrival at the geophones of waves reflected from various geological beds.
t_1, t_2, t_5—Timing lines spaced 0.010 second apart.

It will be helpful for a better understanding of this paper to include a reproduction of a typical seismic record. The seismogram illustrated in Fig. 1 represents the recordation of A time of explosion—the zero from which time is measured; B time of arrival of explosive impulse energy at the uphole geophone ("uphole geophone break"); C arrival time of the refracted energy by the shortest time path at the first and succeeding geophones

in the spread ("first break" of the reflection geophones); D, E, F, G, etc., arrival times of seismic waves reflected from various subsurface geological beds; T_1, T_2, T_3, etc., individual traces of the various geophones or groups of geophones; and t_1, t_2, t_3, etc., the various timing lines spaced 1/100 second apart by a vacuum-tube-driven elinvar tuning fork. On each record is also stamped the various pertinent information in connection with the exploration program and computation. Such a record yields all the times that are required in ordinary reflection seismograph work. These times, together with other information, permit accurate determination of depths to subsurface geological formations which are of significance in petroleum exploration.

II. Requirements

Reference to the seismogram in Fig. 1 will show that the time break A, the uphole geophone break B, and other time events can be estimated accurately to the nearest 1/1000 of a second. As soon, however, as radio transmission is resorted to, the static disturbances, which are always present, and other noise interferences prevent the traces, on one of which are recorded the first two time instants, from being absolutely quiet. These disturbances prevent the distinguishing of these time instants, which produce effects on the trace essentially similar to the interference. Initially, the traces must be quiet and disturbed only by the electrical impulses which it is desired to record.

The "character" of the uphole geophone break should be recorded. Where there is poor wave transmission in the ground, the uphole geophone break may be rounded off instead of sharp and distinct. It is important to pick the instant where the rounded portion first shows on the trace. Occasionally, an iron casing is inserted in the shot hole to prevent its walls from crumbling and stopping up the hole, so the subsequent shots can be made in the same hole. Often when casing is used, an initial or "casing-break" of small magnitude is observed on the uphole geophone trace. If the undistorted character of the uphole geophone voltage impulse is recorded on the trace, there is no difficulty in distinguishing this impulse from that caused by the casing break. This condition precludes the use of some "trigger" device necessary when using amplitude modulation for the uphole break because if this device is made sufficiently sensitive to operate on the first arrival of energy, then extraneous signals will trip this device and the identification will be lost.

If both the voice and the time signals are transmitted over the same radio channel, there must be a marked difference in the magnitude of the two signals or separate channels used for them. In the former case, if the voice signals approach in magnitude those of the time breaks, then the former will trip certain gas triodes and initiate gain and other controls which are required to come into action automatically at the instant, say, of the time-break impulse.

A further requirement is that the radio channel can be switched off and a wire communication channel substituted for it. Time delay due to phase distortion in the two systems should be the same and reduced essentially to zero. This was accomplished satisfactorily and verified by experimental data. The voice signals and the electrical impulses due to the time-break and uphole geophone signals should be the same whether wire or radio transmission was employed. This also was accomplished, and either wire or radio transmission could be used, requiring only installation of appropriate switching arrangements. This paper, however, does not concern itself with the electronic and other equipment developed for the wire channel.

III. Choice of Amplitude and Frequency Modulation

Experiments have been made with both amplitude and frequency modulation for the transmission of speech and time-instant signals. Since so much static interference was present on the communication trace when amplitude modulation was employed in the earlier development, use was made of the time-break impulse to stop the oscillations of a 1000-cycle oscillator signal superimposed on the signal trace. While this provided a fair location of the time break itself, it was impossible to locate the exact beginning of the break of the uphole geophone, particularly in view of the fact that these two time instants frequently were less than 0.01 second apart. The extraneous disturbances were often far in excess of the desired signals, and the former interfered with the first refracted breaks and other portions of the seismogram.

Frequency modulation by these time-instant impulses seemed to offer a worth-while line of attack and there was obtained from the Federal Communications Commission the necessary experimental licenses which would permit the work to be carried out with both frequency and amplitude modulation and a combination of these two types of modulation.

The problems of voice transmission from the shooting to the recording trucks over the radio channel presented certain difficulties. This transmission might be accomplished by use of frequency modulation of the time break over a much wider frequency range than that used for the voice signals. There is a practical limit, however, to this plan in that it may be necessary to swing too far for the time-instant impulses in order to obtain the desired ratio. It seemed best to discard frequency modulation for voice signals in favor of a simpler scheme; namely, the combination of both amplitude and frequency modulation in the transmitter and a special receiver to receive both types of signals. In this scheme, amplitude modulation could well be adopted for the voice signals and frequency modulation for the two time-instant voltage impulses. This choice proved to be entirely satisfactory.

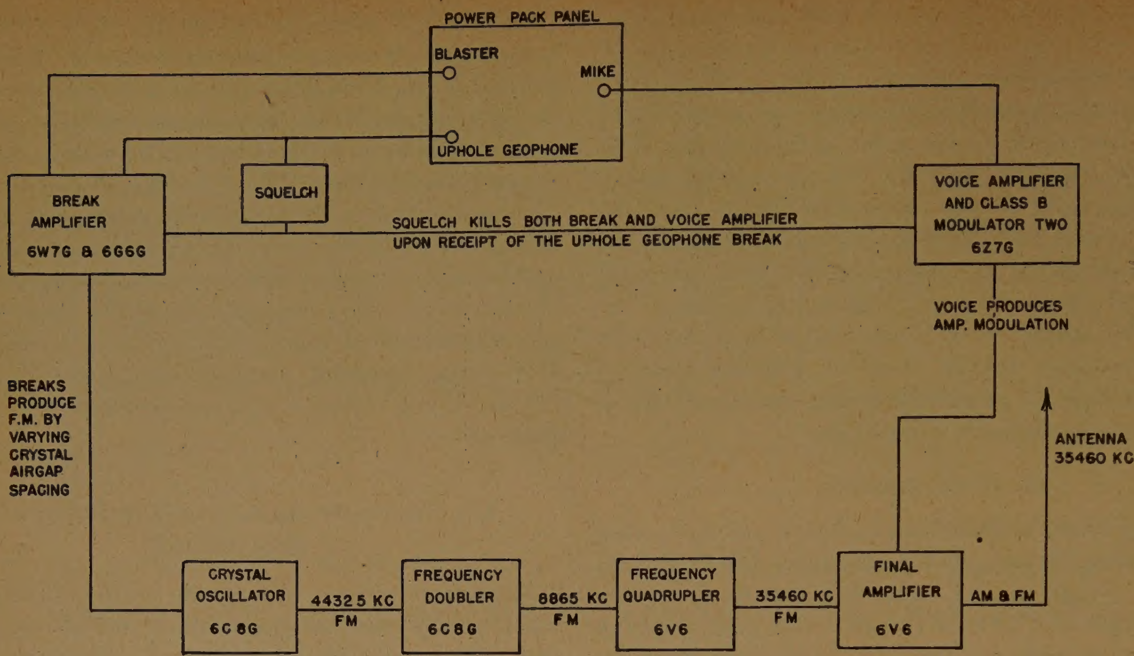


Fig. 2—Block diagram of amplitude- and frequency-modulated seismograph 10-watt transmitter.

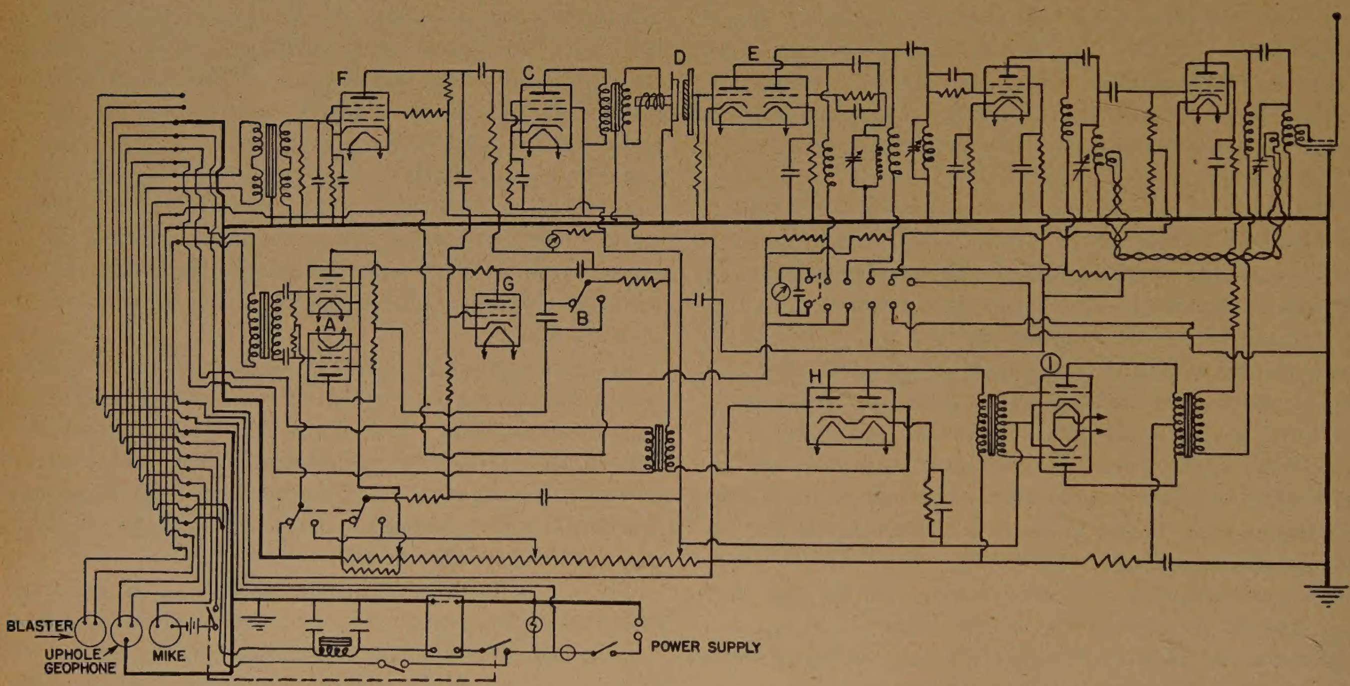


Fig. 3—Circuit diagram of amplitude- and frequency-modulated seismograph transmitter.

A—Push-push gas triodes for time break.
B—Small time-break condenser.
C—Time-break amplifier stage.
D—Crystal for oscillator control and frequency modulation of time break and uphole geophone break.
E—oscillator.
F, *C*—Two-stage amplifier for uphole geophone.
G—Gas triode tube for squelching circuit.
H, *I*—Two-stage voice amplifier.

IV. Transmitter and Associated Equipment

A block diagram of the transmitter with the associated equipment for speech and time-instant impulses is shown in Fig. 2. The circuit diagram of the transmitter is shown in Fig. 3, and a photograph of the assembled unit is shown in Fig. 4.

As indicated in Fig. 2, the associated equipment includes the power pack, blasting circuit, uphole geophone, and voice microphone. The latter three devices are connected into the appropriate receptacles, indicated in Fig. 3, lower left-hand corner. The blasting circuit, in addition to containing the generator and associated equipment for detonating the blasting cap, has incorporated an electrical derivating circuit in order to insure an

electrical impulse sufficiently sharp to permit the determination of the time instant of the explosion to within 1/1000 second.

Time-Break Circuit

The time-break impulse from the blaster is applied to two gas triodes in push-push arrangement, as shown in Fig. 3, at *A*. The first swing (regardless of its direction) of this impulse will "trip" one or the other of these gas triodes. The resulting plate current flows through a small condenser *B*; is amplified by one stage *C*; is used to frequency-modulate at *D*, the crystal-controlled oscillator *E*; again amplified and finally radiated by the 10-watt transmitter. The circuit and block diagrams permit the reader to follow the operation without difficulty.

The purpose of the small condenser *B* is to remove effectively the plate voltage from both these gas triodes so that one and only one of them will be tripped by the time-break impulse. This is accomplished by the flow of the plate current which quickly charges this condenser to absorb the plate voltage. The sharpness and distinctness of such a break is indicated in Fig. 1, at *A*.

Uphole Geophone-Break Circuit

The uphole geophone break is amplified by two stages as shown in Fig. 3, at *F* and *C*. It is then used to frequency-modulate the oscillator, amplified and radiated over the same channel used for the time break. Thus, the amplifier stage *C*, ahead of the frequency-modulating device, and the remaining part of the transmitter are common to both the uphole and time breaks. The distinctness of the uphole geophone break is apparent, as indicated in Fig. 1, at *B*.

These two breaks are close together, often only a few thousandths of a second apart. The use of the gas triodes *A* for the time break and the small condenser *B* associated with them permit the attainment of a time break of short duration, as is essential when it is followed so closely by the uphole geophone break.

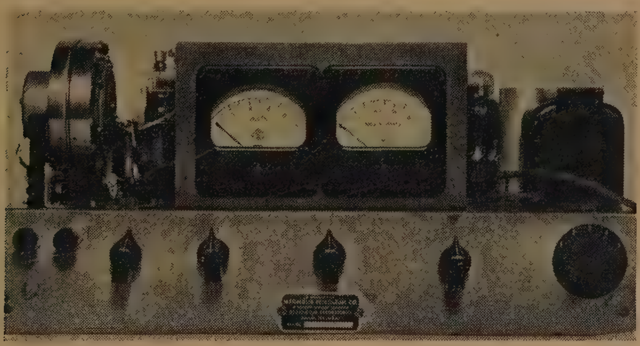


Fig. 4—Photograph of the assembled 10-watt transmitter.

Squelch Circuit

After these impulses are transmitted it is desired to squelch their circuits completely so as to enable the same recording channel to be used for one of the traces of the record. For this purpose, the gas-triode grid of the squelching tube *G* is connected to the output of the first stage *F* of the uphole geophone amplifier in such a manner that its grid is made more negative by the first leg of the uphole geophone impulse. This half of the uphole geophone impulse is transmitted unmodified, as

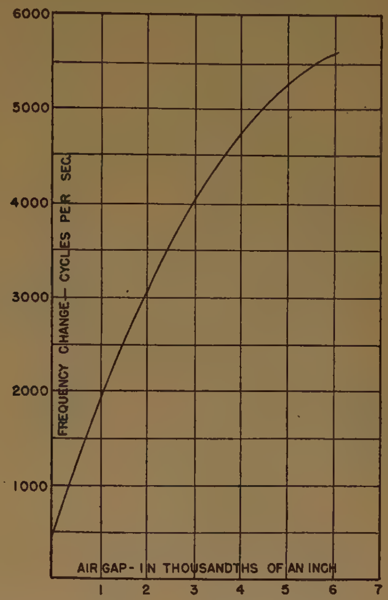


Fig. 5—Frequency-modulated crystal-modulator characteristics. Frequency swing plotted against variation of air gap in frequency-modulated modulator *D*.

desired. The second leg drives this squelch tube *G* grid sufficiently toward zero as to cause it to lose control, and the tube is tripped. The plate current of this tube then biases to cut off the grid of the second tube *C* of the uphole geophone amplifier, thus preventing the transmission of any later impulses from this geophone. This effect takes place practically instantaneously.

Voice-Signal Circuit

The voice-signal circuit can be traced readily, in Figs. 2 and 3, from the microphone through the initial two stages *H* and *I* of amplification, then used for amplitude modulation at the final stage and radiated. The first stage *H* of the voice amplifier shows sections of a 6Z7G in parallel and has its grids connected to the squelch circuit. Thus, when the gas-triode squelch tube *G* is tripped by the uphole geophone the grids of the first tube *H* of the voice amplifier are biased instantly to cut off. In this manner no electrical disturbance at the microphone can be transmitted to interfere with the response of the photographic trace to the desired seismic signals. The second stage *I* of the voice amplifier is likewise a 6Z7G in class B. In order to prevent severe overload to the vibrapack when modulating by voice, a large 80-microfarad 450-volt dry electrolytic condenser is connected to supply the peak demands of class B modulation.

Frequency-Modulating Device

As shown in Fig. 3, the frequency-modulating device *D* is a modified magnetic type of earphone. This modification had to be accomplished with precision in the construction of the air-gap and crystal assembly of the oscillator circuit. In Fig. 5 is shown the curve of the

frequency change in cycles plotted as a function of the air gap length in thousandths of an inch.

It is believed that the block diagram contains sufficient other information to complete the essential description of the transmitter.

intermediate frequency. By means of a one-turn link circuit, Fig. 7, to increase the coupling between the intermediate-frequency primaries and secondaries, it was possible to broaden the response of the intermediate-frequency transformers to provide bandwidths of approximately 30 kilocycles.

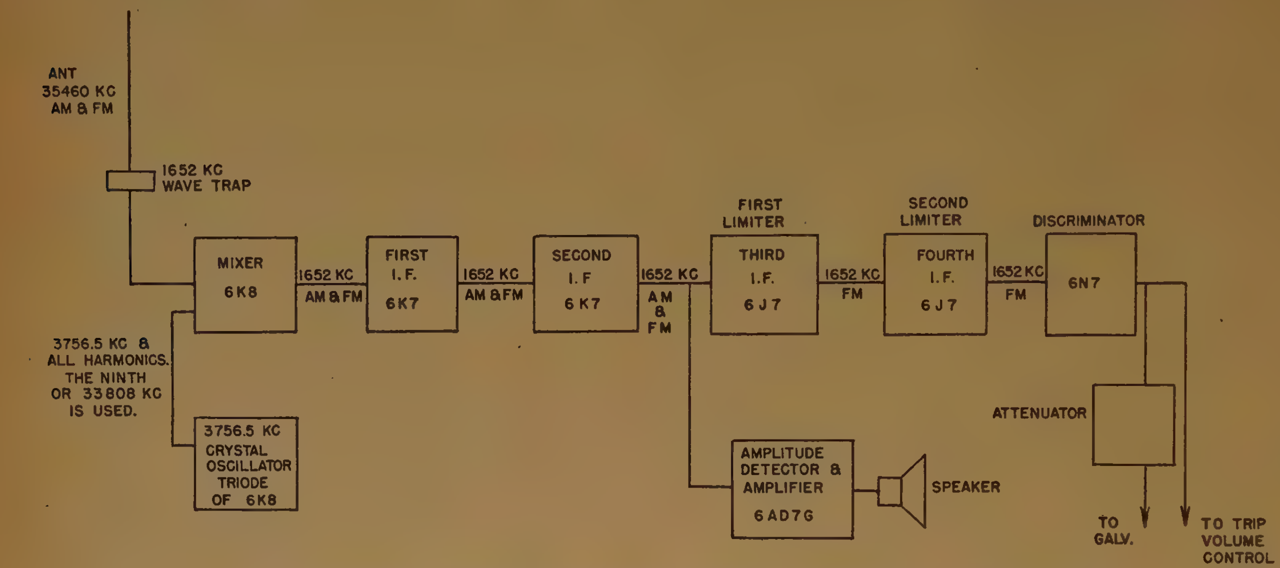


Fig. 6—Block diagram of amplitude-modulation—frequency-modulation seismograph receiver.

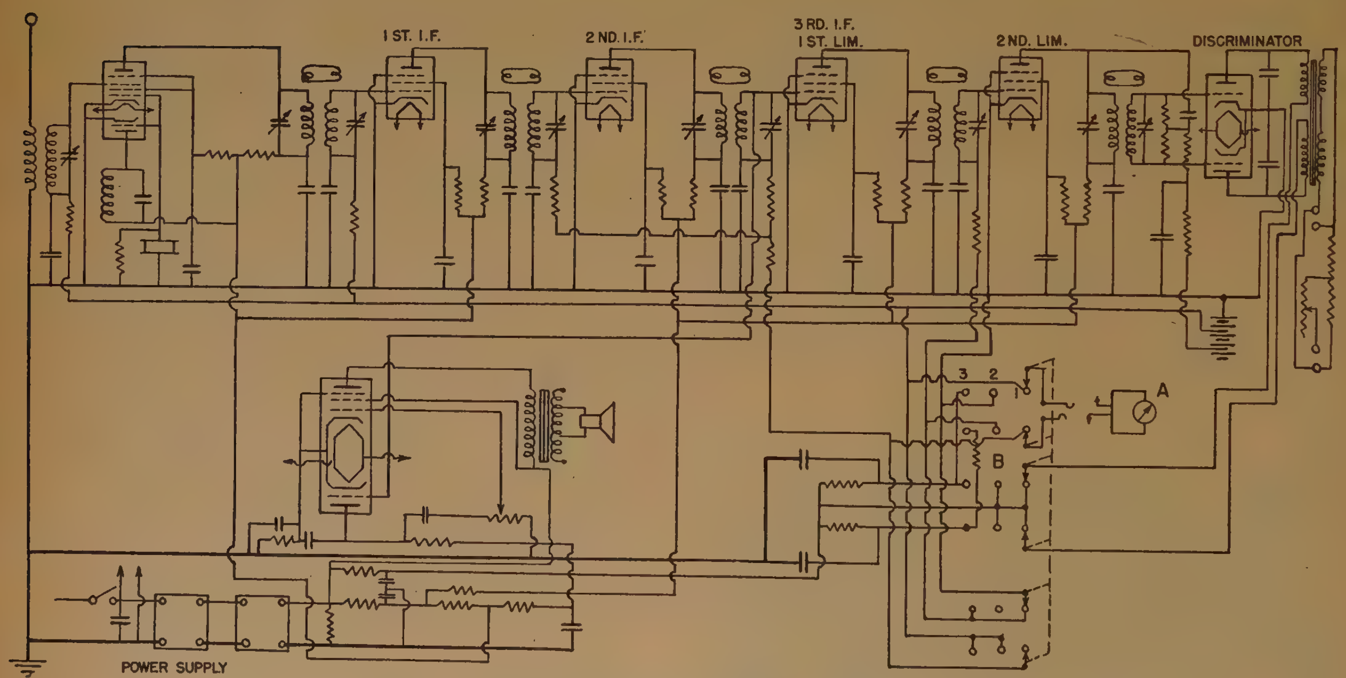


Fig. 7—Circuit diagram of amplitude-modulation—frequency-modulation seismograph receiver.
A—Microammeter for testing and adjustments.
B—Switching arrangements for checking.

V. Receiver

The block and circuit diagrams of the receiver are shown in Figs. 6 and 7, respectively.

As indicated in Fig. 6, the fundamental frequency of the oscillator is 3756.5 kilocycles and is fixed-crystal-controlled. The ninth harmonic is used to give a frequency of 33,808.5 kilocycles. This frequency beats with the incoming 35,460 kilocycles to give a 1652-kilocycle

The 1652-kilocycle wave trap shown in Fig. 6 was provided to avoid interference between the 1652-kilocycle amplitude-modulation transmitter in the recording truck, adjacent to the amplitude-modulation—frequency-modulation receiver. This situation arose from the fact that the regular amplitude-modulation transmitters were assigned to the 1652-kilocycle band.

Comparison of Amplitude-Modulation with Single and Double Limiters in Frequency Modulation

As has been stated, noise interference superimposed on the photographic trace on which are recorded the time-break and uphole geophone, causes indefinite identification of these two time instants. This is illustrated in Fig. 8, which is a sketch of actual photographic traces. The top trace represents an unsuccessful attempt to transmit by amplitude modulation the time and uphole geophone breaks in the midst of severe static interference. The second trace shows where the two time-instant impulses occur, as recorded by frequency modulation with a double or cascade limiter, presently to be described. This type of limiter, together with frequency-modulation transmission, was successful in providing a clear definition of the time break (first break in the trace) and of the later uphole geophone break. Both of these breaks could be distinguished even if they had been close together. Frequency-modulation transmission with one limiter failed to permit successful identification of the time-instant impulses. This is evident in the bottom trace for a single-limiter stage.

Even though frequency modulation with a single-stage limiter is superior to amplitude modulation, nevertheless, the former is quite unsatisfactory for the purpose desired.

Limiter

With respect to the limiter, the traces shown in Fig. 8 indicate that a tolerant attitude cannot be taken with respect to any static interference for the reason that this disturbance is so similar in character to the impulses which are to be transmitted, recorded, and identified. The requirement perhaps is more stringent than encountered in the use of frequency modulation for signal transmission for broadcast purposes.

The limiter circuits tried initially were of the single-stage type and were found inadequate in that enough static still got through to prevent their successful application to the problem at hand. A successful reduction to practice was achieved when a two-stage limiter was devised as shown in Figs. 6 and 7. At the time these experiments were under way, double limiters had not generally been described in the literature, and this work went forward without the benefit which would have otherwise accrued. Therefore, the description of the circuit which follows may vary somewhat from present practices, yet the results obtained were quite satisfactory for seismograph work.

In this device both limiter stages were driven hard to accomplish sufficient noise elimination. It was found that a signal of sufficient amplitude to cause a minimum current of 75 microamperes in the grid circuit of the first limiter was necessary to produce proper limiting. This current produces a $7\frac{1}{2}$-volt drop across the 100,000-ohm grid resistor which in addition to the normal $7\frac{1}{2}$-volt battery bias gave a total of 15 volts bias on the first limiter. For all values of signal causing grid current in the first limiter of 75 microamperes, or greater, the second-limiter grid current was 150 microamperes. Under this condition both limiter tubes were biased beyond plate-current cutoff. This meant that the limiter tubes repeated only portions of the positive half cycles. The tank circuits supplied the remaining portions of each cycle.

This method of limiting proved to be satisfactory in the presence of severe static interference as indicated by the second trace in Fig. 8. This was found to be true

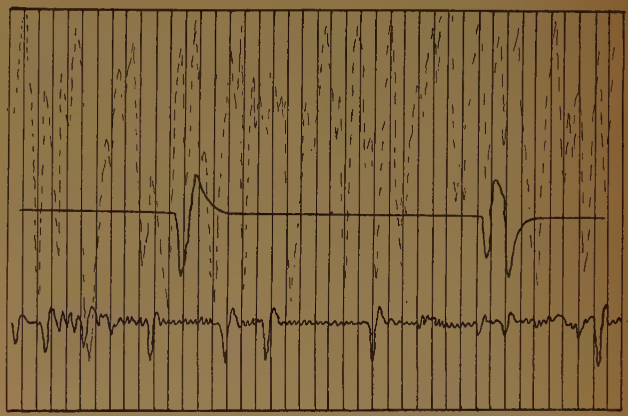


Fig. 8—Tracing of record of time break and uphole geophone.
Upper Trace—Unsuccessful attempt to transmit time instants by amplitude modulation with severe static.
Middle Trace—Successful transmission by frequency modulation with two-stage limiter.
Lower Trace—Unsuccessful transmission by frequency modulation with one-stage limiter.

for the receiver as constructed and illustrated in Figs. 6 and 7 up to the maximum range of approximately 1 mile as required in reflection seismograph exploration.

Discriminator

The discriminator circuit, as shown in Fig. 7, made use of the amplification of the 6N7 tube and also provided for a faked center tap on the discriminator transformer secondary. This circuit was improvised because no standard discriminator transformer was on hand at the time. The operation of this discriminator is basically the same as those described in literature. The performance curve for this discriminator is shown in Fig. 9 and need not be discussed.

Voice-Signal Circuit

In the present system the voice and "breaks" are received over the same radio channel. The time-break impulse is made use of to trip a gas triode whose plate is connected to a circuit to control the amplification of the seismograph amplifiers as a function of time during the recording of the seismic signals. This time-break impulse would have to be of large magnitude relative to the voice signals, if frequency modulation were used for the latter, otherwise these voice signals would trip the volume-control gas triode, a thing to be avoided. Also, extraneous noise signals from the shooting-truck microphone would be received and interfere with the

interpretation of the seismic record. For these reasons, the voice signals were transmitted and received by amplitude rather than by frequency modulation. Thus, the radio system is a combination of amplitude and frequency modulation, the latter being employed for a few hundredths of a second only; while transmitting the time-break and uphole impulses.

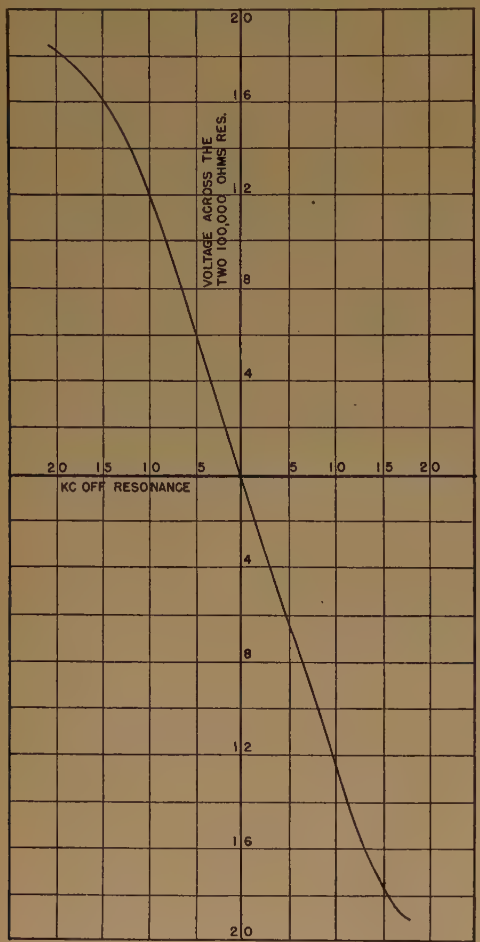


Fig. 9—Performance of discriminator.

In the receiver the voice-amplitude-modulated signals are taken off the intermediate-frequency system just ahead of the first-limiter tube. These signals are detected, amplified, and fed to the loudspeaker as is readily perceived by referring to Figs. 6 and 7.

The voice-amplitude-modulated signals are eliminated by the limiter for the volume levels used. Additionally, the transmitter squelch circuit functions to bias the transmitter voice amplifier to cutoff, as previously explained, as an added precaution. With such an arrangement, the operator does not have to perform any switching operations when the dynamite is being detonated and the seismogram recorded.

Adjustment

In the operation of the receiver (Fig. 7), one 0 to 500 microammeter *A* is used for all the measurements and adjustments. Switching arrangements *B* enable the meter to be inserted in all the circuits which require adjustments. For example, in No. 1 position the meter will indicate the average rectified direct current in the grid circuit of the third intermediate-frequency stage. Tuning of this third intermediate-frequency grid circuit, as well as tuning all the tuned circuits ahead of this point, including the mixer, is indicated by the meter when switched to this position. Additionally, with all such circuits tuned properly, the meter in this position serves as a signal-level indicator. In No. 2 position the meter is switched to the grid circuits of the second limiter to tune the plate circuit of the first limiter and the grid circuit of the second limiter. One might prefer, however, to have the meter in this second position and arrange to indicate all the tuning up to the grid of the second limiter and use the first position for signal-level indication only. The discriminator tuning is indicated by a No. 3 position for the meter.

When the meter is switched to this third position, the two grid circuits, which are metered in positions 1 and 2, are closed. The same operation inserts a 100,000-ohm resistor in each half of the primary of the output transformer. There is inserted a 100,000-ohm resistor in series with the meter by this same operation, converting the meter into a voltmeter. This voltmeter is connected across the two 100,000-ohm resistors in series—one in each half of the primary of the output transformer.

If a constant frequency is now applied to the discriminator and if this frequency is different from that for which the secondary is tuned, an unbalance of the plate voltage results across these two resistors. When the secondary is properly tuned, no unbalance exists in the plate circuits and the voltage thus indicated across the two resistors in series is zero. Tuning of the discriminator is accomplished in this third position as follows:

With the correct or resting frequency applied, the secondary is detuned until a suitable indication is obtained on the meter. With the secondary so detuned, the primary is adjusted for maximum meter reading. Then the secondary is tuned properly by adjusting its tuning condenser for zero reading of the meter. After tuning has been completed, the meter may be switched back to the first position and used, as stated, for the signal-strength indicator.

Correction

It has been brought to the attention of the editors that an error appeared in "Vacuum Capacitors," by G. H. Floyd, which appeared on pages 463 to 470 of the August, 1944, issue of the PROCEEDINGS. In formula 3 on page 466, the figure 1 (one) should be replaced by a capital *I* so that the formula reads $V=I/2\pi fC$.

Some Notes on Superregeneration with Particular Emphasis on Its Possibilities for Frequency Modulation*

HENRY P. KALMUS†, ASSOCIATE I.R.E.

Summary—**The effect of voltage on an oscillatory circuit loaded by a negative resistance is shown and a method for measuring very small voltages is described.[1]**

The superregenerative detector for amplitude modulation possesses limiting action and tends to reject ignition noise. The importance of the shape and frequency of the quench voltage is considered. Selectivity conditions are studied. Selectivity is low for broadcast and high for short-wave frequencies compared with a nonregenerative circuit.

Superregenerative circuits are more susceptible to shot noise than conventional circuits, and this becomes important if preamplifiers are used. Because of back modulation, reradiation can be present even with mixer tubes in front of the superregenerative stage.

An analysis of the frequency spectrum produced by a superregenerative oscillator, when a frequency-modulated signal is applied, is presented. It is shown that the oscillator output consists of a band of frequencies each component of which is deviated in the same amount and direction as the incoming signal. The use of two slope-tuned superregenerative stages in push-pull as a balanced frequency demodulator is described and explained. The superregenerative stage represents a very efficient amplifier for frequency modulation signals.

Two frequency-modulation receivers are described in which the superregnerative tube is used as an amplifier.

I. Principles of Operation of Superregeneration

If we consider an oscillatory circuit with a time constant $\alpha = R/2L$, a resonant frequency f ($\omega = 2\pi f$), a series resistance R (positive or negative), and an initial voltage e_1, the voltage across the circuit will change according to the equation

$$e = e_1 \epsilon^{-\alpha t} \sin \omega t. \qquad (1)$$

This means that the amplitude of the oscillations increases exponentially if R is negative. This is the case if the circuit is regenerated. The equation holds as long as we work in the linear range of the tube; i.e., the whole procedure can be represented by linear differential equations.

The characteristics of the tube are linear if we work with very small amplitudes and negative bias (no grid current). If the amplitudes increase, an end value is soon reached, determined by the voltage or current limitations within the tube.

It is the nature of an exponential function that a given amplitude is reached sooner if the initial voltage e_1 is higher. In an actual tube oscillator, even though no externally applied voltage e_1 is present, the oscillations reach their final amplitude in a finite time. We must, therefore, draw the conclusion that in the absence of an externally applied signal a noise voltage is present which determines the rate of build-up of the oscillations.

In Fig. 1 the regeneration is applied at time 0. The voltage change from zero up to an arbitrary value of $+E$ lies in the linear working range of the tube, so that the curves up to $+E$ may be considered as pure exponential functions.

Now let us assume that an initial voltage of e_a volts exists at the time zero. In this case the voltage E is represented by the equation $E = e_a \epsilon^{-\alpha t_a}$. If the initial voltage is e_b volts, the equation is $E = e_b \epsilon^{-\alpha t_b}$. If e_a, e_b, and t_a are known (from observation in an oscilloscope), t_b can be easily computed from the relation

$$\log_\epsilon (e_a/e_b) = -\alpha \Delta t \quad \text{where} \quad \Delta t = t_b - t_a. \qquad (2)$$

Equation (2) can be used to determine any small initial voltage by means of two other known voltages. Two voltages are required because it is impossible to determine t_a or t_b on the oscilloscope screen. However, it is easy to measure $\Delta t = t_b - t_a$.

Let us assume e_a is 1 volt; the voltage E is reached after an unknown time t_a. However, it is found that when $e_b = 1/10$ volt, voltage E is reached 10^{-2} second later. Now we apply as e_b the unknown voltage and find that E is reached 5×10^{-2} second later. We apply equation (2):

$$\left.\begin{aligned} \log_\epsilon 1/(1/10) &= -\alpha \cdot 10^{-2} \\ \log_\epsilon 1/x &= -\alpha \cdot 5 \cdot 10^{-2} \end{aligned}\right\} \frac{\log_\epsilon 1/x}{\log_\epsilon 10} = 5;$$

$$\log_\epsilon x = -5 \log_\epsilon 10; \qquad \log_\epsilon x = \log_\epsilon 10^{-5};$$

$$\boxed{x = 10^{-5} \text{ volt.}}$$

The initial voltage is, therefore, 10^{-5} volt or 10 microvolts. From the three curves drawn for 1 volt, 0.1 volt, and 10 microvolts, we can see that they are shifted more and more to the left as the initial voltage becomes greater.

If we start and stop the oscillations periodically by applying and removing the regeneration without, however, inserting an initial voltage, we observe Fig. 2 on the oscilloscope. The time intervals Δt_1, between the starting points of the oscillations are equal and determined by the switching-on time of the regeneration. The initial voltage itself is determined by the shot noise in the tube, if the whole arrangement is carefully shielded. The time intervals Δt_2 are *not* equal and depend on the momentary noise voltages at the time the regeneration starts.

When the regeneration stops, the oscillation decays. The shape of the curve is determined by $e = E\epsilon^{-\alpha' t} \sin \omega t$. α' is again the time constant of the circuit. $\alpha' = R'/2L$

* Decimal classification: R163. Original manuscript received by the Institute, September 10, 1943; revised manuscript received, May 29, 1944.

† Zenith Radio Corporation, Chicago, Illinois.

[1] Heinrich Barkhausen, "Lehrbuch der Elektronen-Roehren", vol. 3, S. Hirzel, Leipzig, Germany, 1935.

and R' is now a positive resistance. The amplitude reaches zero, theoretically, after an infinite time. If the interval between two succeeding wave trains is so long that the oscillations have time to decay below the noise level, we obtain a picture of fluctuating wave trains on the oscilloscope, the wave trains starting with random phase and reaching full amplitude at random intervals, determined by the noise.

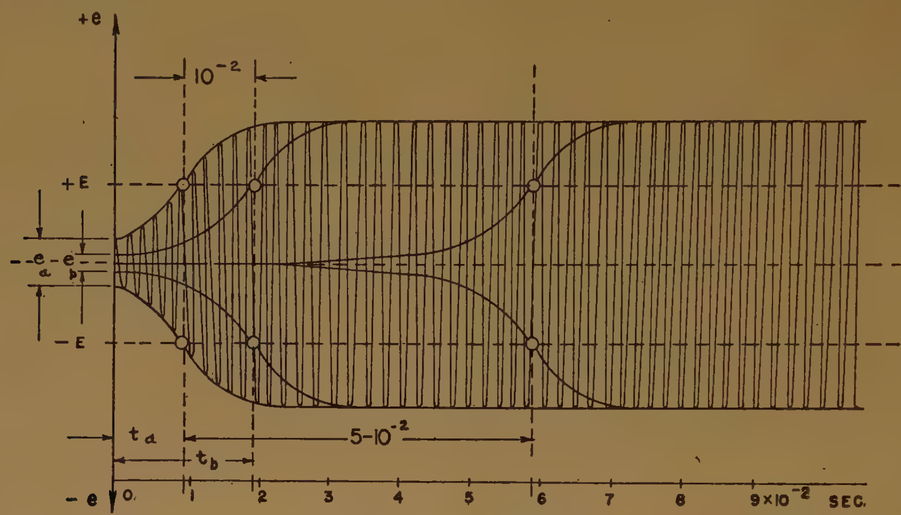


Fig. 1[2]—Build-up of radio-frequency oscillations as a function of the magnitude of the initiating voltage.

If the interval between two regeneration periods is shorter and the oscillations have not sufficient time to decay, a residual voltage will exist at the time when the regeneration starts again. In this case we obtain Fig. 3 on the screen of the oscilloscope. The individual groups reach their maximum now within regular intervals, $\Delta t_2 = \Delta t_2'$. We obtain a standing picture on the screen.

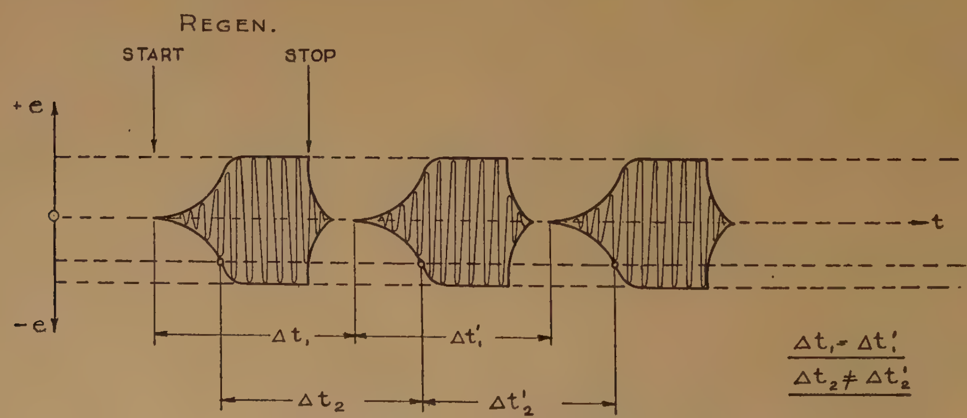


Fig. 2—Incoherent state of superregeneration.

This condition is called *coherent*, while the condition described in Fig. 2 is called *incoherent*.

There is a pronounced difference between the two conditions if we observe the beat note of these oscillations against a continuous wave. In the incoherent case we do not obtain a regular beat note but an irregular noise. The reason is that the oscillations start with a different phase in each regeneration period. A group of wave trains of the same frequency but with random phasing cannot be resolved into a carrier and discrete side frequencies, so that there is no component present which would give a sinusoidal beat note.

In the coherent case the phase of all wave trains is the same. A carrier and discrete side frequencies are present and a sinusoidal beat note can be obtained.

Superregenerative Detection and Impulse Noise Suppression

So far we have seen that it is possible to control the duration of individual wave trains by the amplitude of a small initial voltage of a few microvolts.

Fig. 4 shows a circuit of a simple superregenerative detector. A triode of the 9002 type is connected as generator to an input circuit, which in turn is coupled to an antenna primary. Any small signal in the antenna provides the initial voltage for the circuit, and oscilla-

[2] See footnote ref. 1, Fig. 85, p. 164. Copyright vested in the Alien Property Custodian, 1944, pursuant to law. This figure is reprinted by permission of the Alien Property Custodian in the public interest under license number A-720.

tions are built up until the 50-micromicrofarad condenser C is charged to such a value that the oscillations are stopped. After the condenser has discharged through the 10-megohm resistor R, the oscillations start again. If, therefore, the time constant of C and R is chosen correctly, we get an incoherent condition according to Fig. 3. The amount of regeneration can be controlled by changing the plate voltage with potentiometer P.

$$\Delta t = c \log_\epsilon e_a/e_b, \quad c \text{ is a constant.} \tag{2a}$$

When these wave trains of constant peak amplitude, but variable length, are applied to a rectifier, the average rectified current varies with the length of the wave trains. Since the length of the wave trains follows the amplitude modulation of the original signal the rectified current in the detector reproduces the amplitude modulation. Fig. 5(c) shows the audio output of the rectifier.

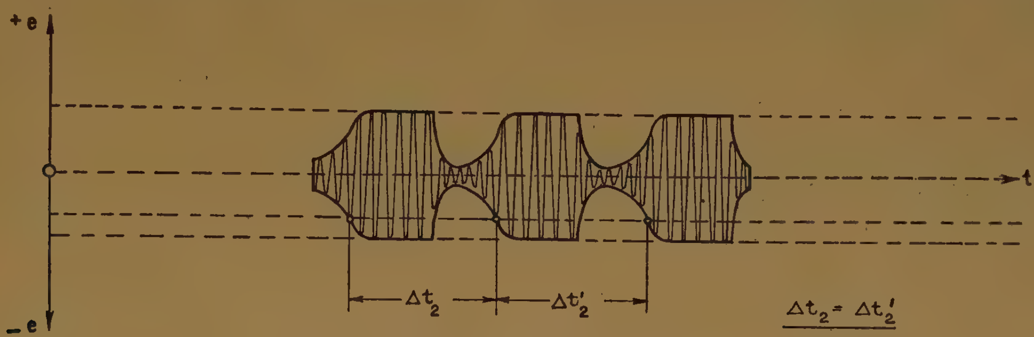


Fig. 3—Coherent state of superregeneration.

Now, if an unmodulated wave is received by the antenna, it will serve to start each individual wave train. The phase of all wave trains, therefore, is controlled by the incoming signal, and we can resolve the groups of oscillations into a carrier and side frequencies. The quench frequency, that is, the frequency at which the oscillations are started and stopped, is determined by the time constant CR and by the degree of feedback.

The position and amplitudes of these side frequencies will be described later when the behavior for a frequency-modulated signal is considered.

The tube in Fig. 4 works simultaneously as a rectifier, and the plate current therefore contains the modulation of the incoming signal. This audio output is independent of the average signal amplitude, and depends only on the relative change of the signal; i.e., on the percentage modulation. If e_a denotes the maximum and e_b the minimum value of a modulated carrier, $\Delta t = c \log_\epsilon e_a/e_b$. If the signal is increased k times, we obtain $\Delta t = c \log_\epsilon ke_a/ke_b$, which is the same as for the original signal. We obtain, therefore, an ideal automatic-volume-control action.

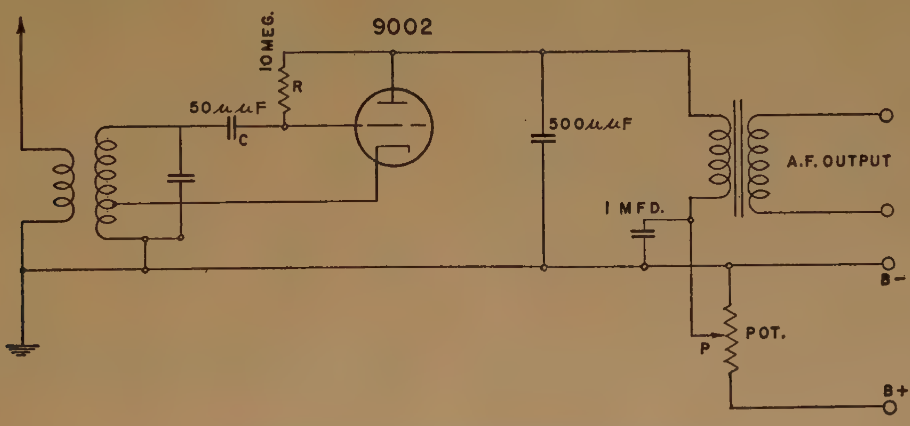


Fig. 4—Circuit of a simple superregenerative detector.

In the event that the incoming signal is amplitude-modulated, the building-up curves will shift from right to left if the signal amplitude increases. Fig. 5(a) shows the incoming modulated wave, and Fig. 5(b) the oscillations after the superregenerative stage. It is clearly seen that the building-up curves a, b, c, and d are located farther to the left when the signal at the switching-on time of the regeneration is larger, and farther to the right if the signal is smaller. This shift in the wave follows (2). In another form, this equation is

Equation (2a) explains, furthermore, the fact that a superregeneration receiver is much less susceptible to ignition noise than an ordinary receiver. Assuming that a pulse ten times larger than the incoming signal arrives just before the regeneration is switched on, we have the relation $\Delta t = c \log_\epsilon 10 = c \times 2.3$. The rectified noise voltage after the superregenerative stage is only 2.3 times as large as the signal, instead of ten times as in an ordinary receiver. If the ratio is 100, the improvement is even more pronounced: $\Delta t = c \log_\epsilon 100 = c \times 4.6$.

An additional muting effect for ignition noise is based on its statistical distribution. Pulses reaching the circuit while it is oscillating with maximum amplitude have no effect on the circuit.

Sensitivity and Selectivity

A simple superregeneration receiver, as in Fig. 4, with a single audio stage, has a sensitivity of 2 microvolts antenna voltage. The sensitivity at the grid of the tube is smaller (about 10 microvolts), and the step-up ratio

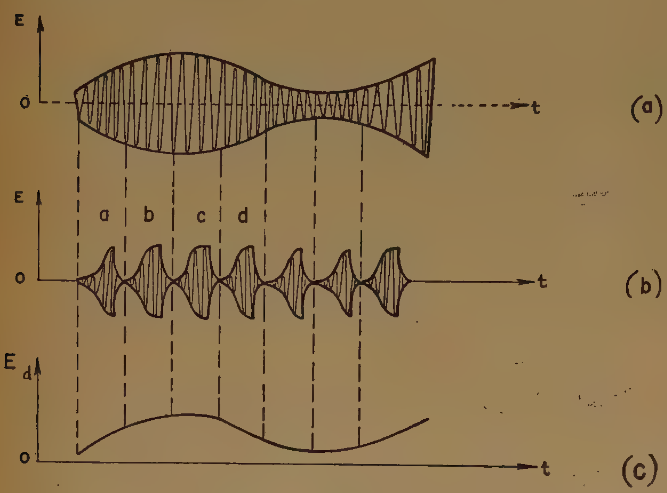


Fig. 5—Incoming amplitude-modulated radio-frequency wave, resultant oscillations in the superregenerative detector, and demodulated output.

of the antenna transformer makes up for the difference. The term sensitivity should be understood as noise-muting sensitivity.

Any small initial voltage across the superregenerative circuit is capable of driving the superregenerative tube into maximum oscillation. This explains the high gain; but it is also the reason why the shot-noise pulses produce maximum audio output when the antenna signal is smaller than 2 microvolts. If the antenna signal is increased, it "takes over" and controls the phase of the individual wave trains, and the noise gradually disappears. An input signal of 5 microvolts suppresses the noise entirely.

The audio output contains the quench frequency, which therefore has to be beyond the audible range. In order to obtain good sensitivity, a certain ratio between signal and quench frequency should be maintained. Apparently, with too high a quench frequency, the oscillation is not given enough time to build up to maximum amplitude.

The circuit in Fig. 4 produces its own quench voltage. It is, of course, possible to employ a separate tube and to quench the superregenerative stage externally and we can choose any shape of quench envelope, sinusoidal, triangular, or rectangular. Experiments have shown that a certain form of trapezoidal envelope is most efficient, and this is just the shape we obtain in a self-quenched circuit. Furthermore, it is necessary to maintain a certain ratio between regeneration and quenching voltage, and this ratio adjusts itself automatically to its optimum value if self-quenching is employed.

The optimum quench frequency for a signal of 45 megacycles is 100 kilocycles; for 75 megacycles, about 200 kilocycles. We can see now that superregeneration can be used with greatest advantage at high frequencies, because for medium frequencies the optimum quench frequency would have to be in the audible range.

If we study the selectivity of a simple superregeneration receiver (Fig. 4), we have to consider the conditions in the circuit not only in the regeneration period, but in the "dead" interval as well. The incoming signal produces a certain amplitude across the circuit during the dead period, and this amplitude represents the initial voltage for the regeneration interval. The Q of the circuit in the dead period determines the selectivity during this interval. Let this be called Q_d.

Although the circuit will not respond to the incoming signal throughout most of the regeneration period Q_r, the Q in the regenerative interval has a certain influence on the selectivity. The more slowly the oscillations are able to build up in the beginning of the regeneration period, the greater will be the influence of the signal on the circuit. Q_r is negative (regenerative), and the larger its numerical value, the higher the selectivity.

Let us consider a circuit to which we gradually apply more and more regeneration. First, it has a definite Q, determined by L, C, and R. $Q=\sqrt{L/C}/R$. If we increase the regeneration, the Q goes up and becomes positively infinite at a certain time. Now, the Q jumps from positive to negative infinity and remains negative during the build-up period. Slow build-up corresponds to high numerical values of this negative Q. Fig. 6 shows these conditions. As the amplitude grows, the signal soon loses its influence on the circuit.

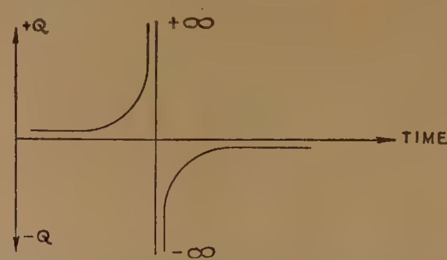


Fig. 6—Variation in Q prior to and following initiation of self-oscillation. At the inception of oscillation the value of Q is $+$infinity.

The resultant over-all selectivity of the circuit can be represented by a hypothetical value Q_h which would be the Q of a simple nonregenerative circuit with the same selectivity.

In order to determine the amount by which Q_d and Q_r contribute to Q_h we have to consider the time interval over which they are active during each quench cycle. Assuming a given quench frequency, more cycles of the oscillator are present for a higher carrier frequency. For the broadcast range, if we want to keep the quench frequency superaudible, the dead interval as well as the starting period contains only a few cycles. Q_d must

be low to keep the oscillations incoherent; and regeneration must be high to effect a fast start, corresponding to a low numerical value of Q_r as mentioned before. The period during which Q_r passes through infinity is extremely short, so that a low value of Q_h should reasonably be expected.

If the carrier frequency is in the short-wave range, the dead interval as well as the starting period may contain many more cycles. A much smaller frequency deviation of the incoming signal will throw it out of phase, so that the effective selectivity should be much higher. This is equivalent to stating that Q_r and Q_d are higher, resulting in a higher Q_h.

Experience shows indeed that Q increases with frequency; in comparison with a simple nonregenerative circuit Q becomes smaller in the broadcast range, but is increased (up to 1000 for 60 megacycles) for high frequencies. A receiver, as shown in Fig. 4 is, therefore, not only much more sensitive, but much more selective as well, compared with a single-circuit receiver without superregeneration for high frequencies.

II. Signal-to-Noise Ratio Considerations in Superregenerative Circuits

The receiver of Fig. 4 has two pronounced disadvantages: reradiation into the antenna, and frequency instability due to the fact that any detuning of the antenna detunes the superregenerative circuit. It is obvious that a radio-frequency stage can be used between the antenna and the superregenerative circuit to overcome these drawbacks. However, although a radio-frequency stage removes the above difficulties, it introduces a new disadvantage. In any radio-frequency stage the plate current produces a random-shot-noise spectrum. The incoming carrier heterodynes the noise bands on both sides and makes the noise much more audible than if the carrier were not present. If there is only one carrier, the noise is proportional to the pass band of the receiver.

As explained in Section I, a superregenerative stage splits up the incoming signal into a number of bands spaced by the quench frequency. All these bands are modulated by the respective noise bands. Therefore, the noise is proportional to the audio pass band of the receiver multiplied by the number of bands produced by the quench frequency.

It will be noted that ignition noise has been treated differently from random shot noise. Previously, we treated the superregenerative oscillations as wave trains. Now we look at them as many continuous-wave bands. There is a distinct difference between ignition and random shot noise which justifies the different treatment of the two problems. The ignition pulses follow each other with rather long time intervals, and their amplitudes exceed the signal voltage considerably. Random shot noise is comparable in amplitude with the signal and is present continuously. Therefore, while shot noise can be treated as a continuous spectrum of carriers, the influence of an ignition pulse does not extend into the following quench period. It is therefore reasonable to consider its influence on a single wave train only.

If we employ a radio-frequency stage in front of the superregenerative circuit, the muting sensitivity, that is the signal required to suppress fluctuation noise, measured at the grid circuit of the radio-frequency tube is not higher than the sensitivity of the superregenerative stage itself. As there are about ten bands within the response envelope of the superregenerative stage, a large amount of shot noise is introduced, which accounts for the drop in muting sensitivity.

In order to investigate experimentally the effect of the plate current on the sensitivity, curves were taken which represent the gain of a radio-frequency tube (7V7 type) followed by an ordinary (curve G) and superregenerative (curve S) detector as shown in Fig. 7. As has been

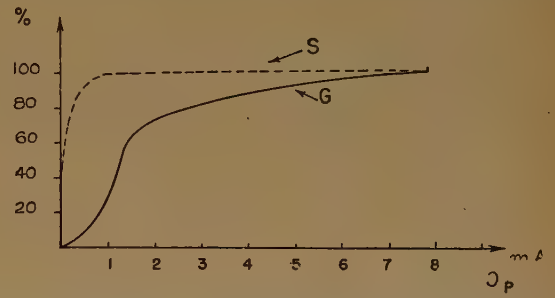


Fig. 7—Comparison of muting sensitivities of receivers equipped with radio-frequency amplifier, with and without superregenerative detector.

stated before, sensitivity in connection with superregeneration means noise-muting sensitivity. Comparing the two curves, we see that we reach maximum sensitivity after a superregenerative stage with a plate current of only 0.5 milliampere, whereas after an ordinary stage that sensitivity rises until the plate current is 8 milliamperes. The curves do not show absolute values and indicate only percentage of maximum noise-muting sensitivity versus plate current. The gain of screen-grid tubes is $g_m \times \omega L \times Q$; g_m rises with the plate current and we can, therefore, see that the higher gain of the tube is canceled by the increase of random shot noise if the tube is used before a superregenerative stage.

It is, therefore, of no use to employ a high-current, high-gain preamplifier tube ahead of a superregenerative stage if the input impedance for the signal frequency is low. A tube of the 9001, 9003, or 6AK5 type is better since the higher input impedance permits a higher antenna gain and hence a better signal-to-noise ratio, the tube noise being the main noise component. The importance of antenna gain is well known for any conventional receiver and is not confined to a superregenerative type; it is however of greater importance with a superregenerative stage because of the greater susceptibility to fluctuation noise. The input impedance of the 9001 and 9003 tubes was measured at 45 megacycles. It is about 50,000 ohms, and an antenna gain of 6 can easily be obtained.

In a superregeneration receiver with a radio-frequency tube reradiation and detuning are avoided only if the superregenerative stage is entirely shielded. If we wish to avoid the shielding, we have to use a mixer and local oscillator instead of a simple radio-frequency stage.

In Fig. 8, a high-frequency pentode tube is used as a mixer tube. The signal frequency is 45 megacycles. The oscillator frequency, produced by a triode, is 25 megacycles. The intermediate and superregenerative frequency is 20 megacycles. Offhand we should think that no reradiation can be present with this circuit. However, if no special precautions are taken, we still observe reradiation at 45 megacycles. The superregenerative stage produces very high amplitudes of about 15 volts at 20 megacycles. These 20-megacycle voltages appear at the plate of the mixer tube.

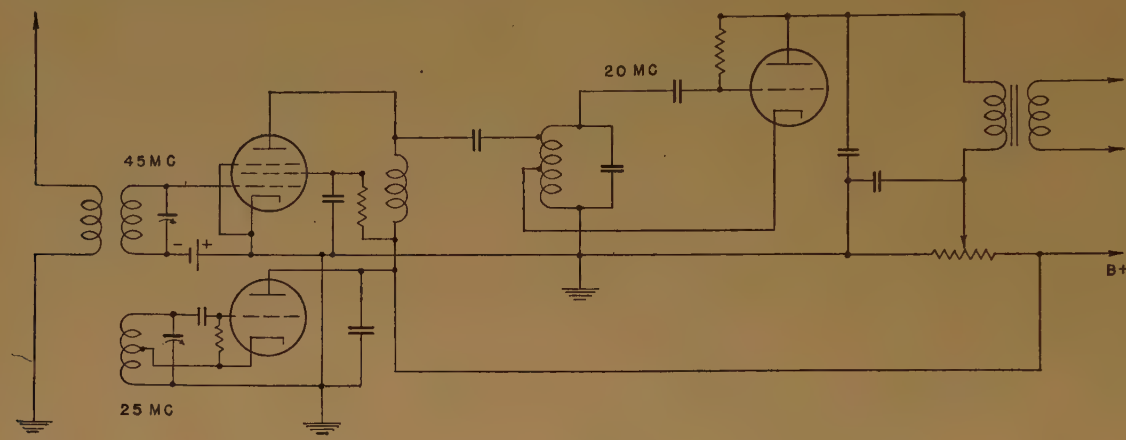


Fig. 8—Receiver with mixer input stage followed by superregenerative detector operating at the difference frequency.

Assuming that the grid-plate capacitance of the mixer tube is 1/100 micromicrofarad and the tuning capacity of the input circuit is 50 micromicrofarads, we have a voltage divider (Fig. 9). If the inductance is so chosen that it tunes to 45 megacycles with 50 micromicrofarads, the impedance of the grid circuit for 20 megacycles will be 40 ohms, while the plate-grid capacitance of 1/100 micromicrofarad represents 800,000 ohms at this frequency. The voltage ratio is, therefore, 20,000:1, and with a plate voltage of 15 volts we obtain 750 microvolts at the grid. This voltage is coupled into the oscillator circuit, and we obtain mixing action in the oscillator tube. Hence, the oscillator tube mixes 20 megacycles with 25 megacycles, and we again obtain 45 megacycles which is reradiated by the antenna.

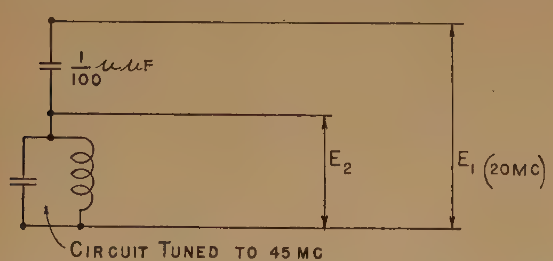


Fig. 9—Circuit showing how oscillations of the superregenerative detector may be coupled back to the mixer input circuit.

In order to prevent this "back modulation," it is necessary to avoid any coupling between the superregenerative circuit and the input circuits. Furthermore, it is necessary to employ a mixer tube with extremely low plate-grid capacitance. If the requirements are very strict, neutralization of the plate-grid capacitance may be employed.

III. Application of Superregeneration to Frequency Modulation

A further analysis of the frequency spectrum produced by a superregenerative oscillator is necessary for an understanding of its behavior when a frequency-modulated signal is applied.

Let us look first at the frequency spectrum produced by a completely quenched, or incoherent oscillator in the absence of an incoming signal. Referring to Fig. 2, we may write for the oscillation during a regeneration period

$$W = A(t) \sin \left[2\pi f_1 t + \phi(t)\right] \tag{3}$$

where f_1 is the frequency of the oscillator, $A(t)$ is the amplitude function corresponding to the oscillation envelope, and $\phi(t)$ is a phase angle which must be added to $2\pi f_1 t$ in order to obtain the actual phase of the wave train W. For a completely quenched oscillation, $\phi(t)$ is a nonperiodic function of time since it is determined only by the random-noise pulses existing at the start of the oscillation. Its frequency spectrum is continuous, and contains no discrete-frequency components. Hence the frequency spectrum of the oscillator output, since it is phase modulated by $\phi(t)$, is also continuous, and contains no discrete components.

If $A(t)$ does not go to zero at any time in the quench cycle, the oscillation is coherent, and $\phi(t)$ disappears. The expression then becomes that of a carrier of frequency f_1, amplitude-modulated by $A(t)$, which is periodic with the quench frequency F. The spectrum of W then contains a number of discrete components at the frequencies f_1, $f_1 \pm F$, $f_1 \pm 2F$, etc.

The same situation prevails if the oscillator is completely quenched, but with its initial phase determined by an injected signal at the frequency f_1. Again $\phi(t)$ is zero, and the output spectrum is the same as that just described for the coherent state.

Now let us suppose that the injected signal has a frequency f_2 different from f_1. If the oscillator is incoherent the phase angle of the oscillator voltage changes during each active period at the rate $2\pi f_1$, where f_1 is the natural frequency of the oscillator. The initial phase angle at the beginning of each active period however is the same as that of the incoming wave at the frequency f_2 at that moment. Now, if we try to represent this sequence of wave trains in the form of (3), we shall find (Appendix) that in the general case, where $f_1 - f_2$ is incommensurable with F, $\phi(t)$ is not periodic. However, if we choose f_2 as the carrier frequency instead of f_1, so that (3) becomes

$$W = A(t) \sin [2\pi f_2 t + \theta(t)], \tag{4}$$

the new function $\theta(t)$ is periodic with the quench frequency. The form of this function must be such that it is zero at the start of each wave train and that it changes the instantaneous frequency from f_2 to f_1 throughout the wave-train duration. Fig. 10(a) shows a series of wave trains with the natural frequency f_1 in which, for simplicity, the amplitude modulation is taken as a square wave with equal live and dead periods (Fig. 10(b)). The form of the phase-modulation function $\theta(t)$, if the wave-trains are controlled by a carrier frequency f_2, is shown in Fig. 10(c); $\theta(t)$ changes linearly with time within each wave-train period from zero to $\pi(f_1 - f_2)/F$.

This recurring phase change and the amplitude modulation due to the quench action are both periodic with the quench frequency and hence produce side frequencies separated from the carrier by the quench frequency and its multiples. Since the carrier frequency is f_2, the output spectrum consists of discrete components at f_2, $f_2 \pm F$, $f_2 \pm 2F$, etc. Although the instantaneous frequency of the output during any regeneration period is still f_1, this frequency has disappeared from the spectrum because of the periodic synchronization of the phase with that of the incoming frequency.

When the incoming frequency is varied each component of the oscillator-output spectrum is shifted in the same direction and amount so that the change in the incoming frequency is found also in each component of the oscillator spectrum. This apparent synchronization of a superregenerative oscillator by an incoming signal has been mentioned by other workers, and has been treated as though it were similar to the synchronization of an unquenched oscillator. The above treatment, however, indicates an entirely different mechanism. When an unquenched oscillator is synchronized by an external voltage, the phase velocity of the oscillation vector is varied during a part of *each* oscillation cycle, so that the average phase velocity over the entire cycle agrees with that of the external voltage. In the case of a quenched oscillator, as we have seen, the oscillator frequency is not controlled except at the start of oscillation, when the small amplitude of oscillation makes it possible for a very small external voltage to force synchronization for a few cycles. There is a great difference in sensitivity in the two cases; an unquenched oscillation with an amplitude of a volt may require an external synchronizing voltage of a tenth of a volt for synchronization over a given frequency range, while a quenched oscillator with the same amplitude may require only 10 microvolts.

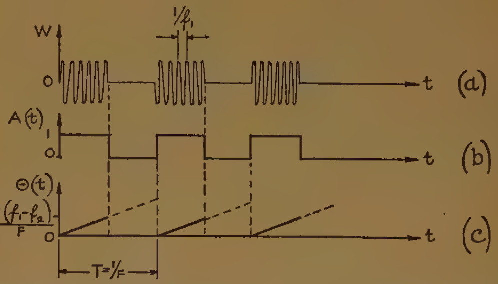


Fig. 10—Simplified representation (a) of the output of the superregenerative oscillator as a modulated carrier wave, showing (b) the amplitude modulation due to the quench action, and (c) the phase modulation if the incoming frequency differs from the natural frequency.

While the amplitudes of the individual side frequencies can be found by lengthy series expansions, an experimental determination is the only practical method. Measurements of the side-frequency amplitudes for a typical superregenerative oscillator have shown that the shape of the amplitude-frequency-distribution curve is independent of the frequency of the incoming signal. That is, the largest amplitudes are always found near the center of the resonance curve of the oscillator tank circuit (Fig. 11). All that happens when the signal is detuned from the center of the resonance curve is that the amplitudes of all the side frequencies are reduced proportionately, while the shape of the amplitude-frequency curve remains the same.

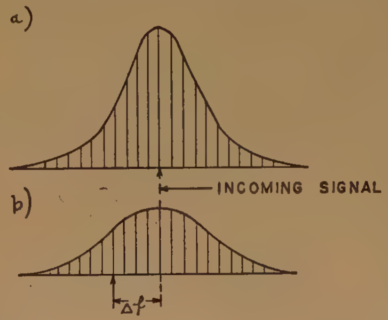


Fig. 11—Amplitude-response characteristic of superregenerative detector (a) tuned to carrier as for amplitude-modulation detection and (b) tuned to one side of the carrier as for frequency-modulation detection.

In Fig. 11(a) the incoming signal has the same frequency as the resonant frequency of the superregenerative circuit. In Fig. 11(b) the incoming signal is detuned by Δf.

The above conclusions were verified by measurement of the side frequencies and their amplitude distribution in a circuit similar to that shown in Fig. 8.

Superregenerative Stage as Frequency-Modulation Demodulator

A circuit as described in Figs. 4 and 8 can be used to detect frequency-modulated signals. The superregenerative circuit is detuned from the incoming frequency, and conversion of frequency modulation into amplitude modulation is obtained along the slope of the resonance envelope of the superregenerative stage. However, by doing this, no complete limiting action is obtained because, as mentioned before, a superregenerative stage demodulates an amplitude-modulated signal. This is the reason why slope detection, although it gives good demodulation, is not advisable. If used, it is necessary to employ two superregenerative stages in push-pull. The two stages can be used slightly detuned, but interaction between the circuits makes elaborate shielding and a special preamplifier tube necessary.

Another way to avoid interaction is by heterodyning the carrier. For instance, if the incoming frequency is 45 megacycles, the fixed oscillator works at 10 megacycles. We obtain two intermediate frequencies, one at 35 megacycles and the other at 55 megacycles. If both are fed into slope-tuned superregenerative stages and the audio output is combined in a push-pull circuit, noise-free frequency-modulation reception may be obtained. The tuning of the oscillator and both superregenerative stages has to be extremely stable and this arrangement does not seem to be practicable.

Superregenerative Stage as Frequency-Modulation Amplifier

The superregenerative stage has generally been used as a detector. However, by using it as an amplifier a very efficient frequency-modulation receiver can be designed. As shown before, an input voltage of 10 microvolts on the grid of the superregenerative tube is strong enough to synchronize the oscillator. As the incoming frequency varies, each component of the oscillator spectrum varies by the same amount.

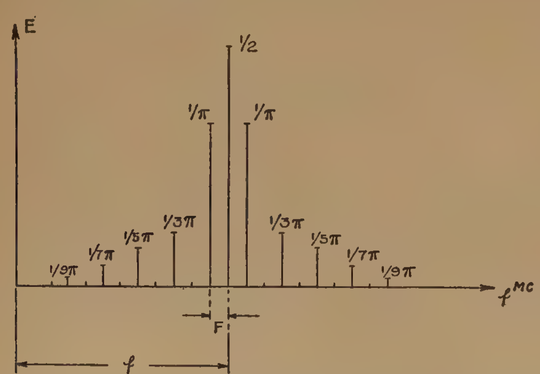


Fig. 12—Side-frequency distribution of the wave shown in Fig. 10, if $f_1 = f_2$.

There are two ways to obtain frequency-modulation detection:

1. The total spectrum is transposed by a mixer tube to lower frequencies, centered say, at 8.3 megacycles. If the quench frequency is 100 kilocycles, we obtain frequencies at 8, 8.1, 8.2, 8.3, 8.4, 8.5, etc., megacycles. The mixer tube works into a rather narrow band-pass filter, which responds to one of the bands only, but is broad enough to cover the maximum frequency-modulation deviation. This single band is fed through a limiter stage into any kind of frequency discriminator.

To find out whether, and how, we can obtain distortion-free demodulation we have to consider again the amplitude-frequency distribution of the spectrum. If we assume $f_1 = f_2$, that is, carrier and oscillator frequency equal, we can neglect side frequencies produced by phase modulation. For a square wave as shown in Fig. 10, where the "working" and "dead" intervals are equal, we obtain a frequency spectrum as shown in Fig. 12. Components differing from f_1 by even multiples of F do not exist in the spectrum.

Under more general conditions all possible side frequencies will exist. Fig. 13 illustrates this case. In the

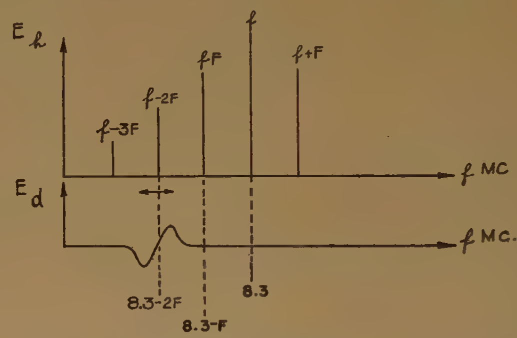


Fig. 13—Discriminator applied to one of the side frequencies of Fig. 12.

lower part of the picture a discriminator characteristic is shown. Now, if the side frequency (f–$2F$), after being transposed to a lower frequency, is fed into a discriminator we obtain demodulation. This demodulation is distortion-free only if no other band enters the discriminator range. The quench frequency, therefore, has to be higher than the maximum frequency-modulation deviation.

2. Another interesting possibility for frequency demodulation is to use a multiresponse frequency-modulation discriminator, which responds to all side frequencies at once. A multiresonant frequency-modulation discriminator might be designed by the use of a line, resonant at the quench frequency. Such a line shows resonant points separated by the quench frequency and it will therefore operate with a number of side frequencies in the same way a single-tuned circuit operates with one component. Two lines can be used in push-pull to form a balanced discriminator.

IV. Receiver Design

Fig. 14 shows a frequency-modulation receiver using a superregenerative stage as an amplifier. The 45-megacycle signal passes an antenna circuit, a radio-frequency tube, and is impressed on a superregenerative stage. The

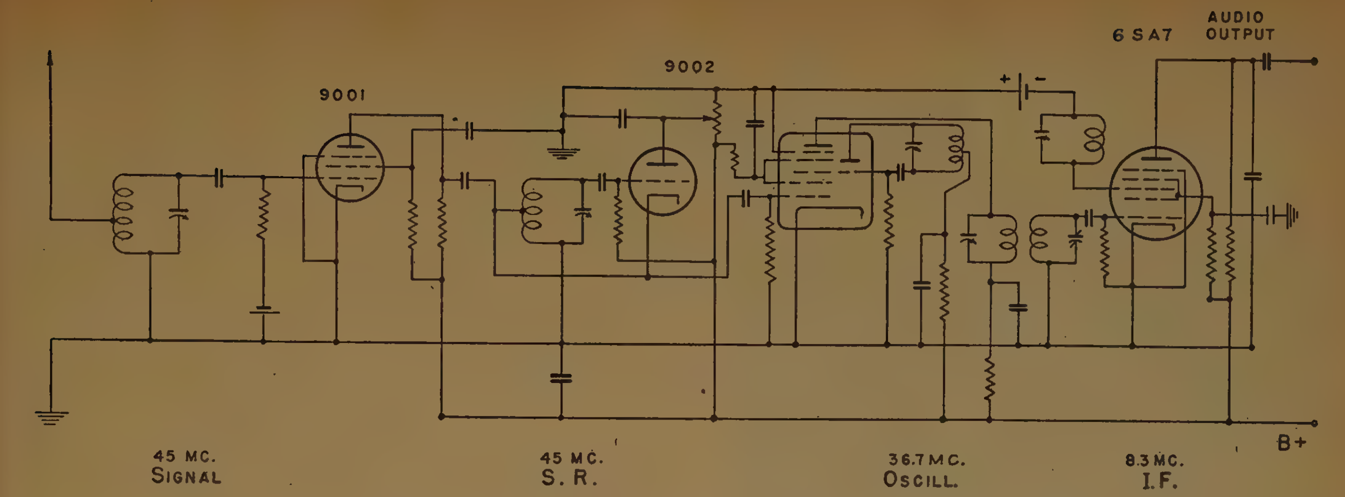


Fig. 14—Frequency-modulation receiver using a superregenerative amplifier stage.

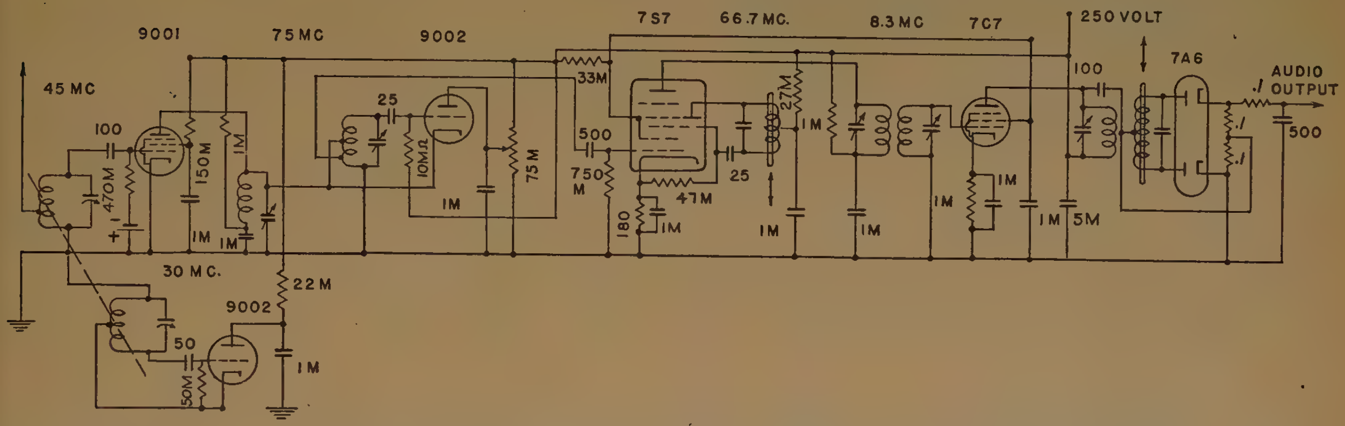


Fig. 15—Frequency-modulation receiver for wider band frequency modulation using a superregenerative amplifier stage.

superregenerative tube is self-quenched and splits the signal into a number of side frequencies. The whole frequency group is transposed to a spectrum in the 8.3-megacycle range. One of the frequencies is then selected by the double-tuned intermediate-frequency transformer and controls a space-charge discriminator.[3]

This discriminator supplies enough audio power for a headset. Only one additional audio tube is necessary to control a speaker. The receiver is extremely free from ignition noise because of cascade limiting in three succeeding stages: The superregenerative stage works as a limiter due to its logarithmic characteristic. The signal produced by the superregenerative tube has an amplitude of about 3 volts (for one band) and is clipped by the mixer tube. The space-charge discinimator works simultaneously as a third limiter. The noise-muting sensitivity is 2 microvolts; the selectivity is rather poor, being determined by the antenna and superregenerative circuits. It is sufficient to separate the three Chicago frequency-modulation stations, working at 45.1, 45.9, and 46.7 megacycles.

As stated above, the quench frequency has to be higher than the maximum deviation in order to obtain distortion-free demodulation. Now, it evidently becomes easier to produce a high quench frequency as the frequency in the superregenerative stage is increased. Tests have shown that, with a 9002 tube, it is barely possible to obtain the quench frequency of 150–200 kilocycles required for 150-kilocycle deviation at a signal frequency of 45 kilocycles.

If the superregenerative tube works at 75 megacycles instead of 45 megacycles, a quench frequency of 200 kilocycles can easily be produced. Fig. 15 shows the circuit diagram of a receiver employing 75-megacycle superregeneration. The incoming signal (45 megacycles) passes the input transformer and enters the first mixer tube (9001). The oscillator works at 30 megacycles. The resultant first intermediate frequency (75 megacycles) passes a band-pass filter and is impressed on the superregenerative stage.

The superregenerative tube again produces a band spectrum which is heterodyned in the second mixer and transposed to an 8.3-megacycle spectrum. One single band is selected. As it passes a combined amplifier limiter it is demodulated in a double-diode discriminator. (A space-charge discriminator can also be employed.)

In an ordinary frequency-modulation receiver the space-charge discriminator has the disadvantage that it is not balanced. A double-diode discriminator does

[3] United States Patent, No. 2,233,706.

not respond to amplitude modulation if the incoming signal is at mid-frequency. A space-charge discriminator, although working simultaneously as an additional limiter, responds in a certain degree to amplitude modulation. In a receiver with a superregenerative stage as amplifier, however, a space-charge discriminator works remarkably well owing to the good limiting action of the superregenerative stage, which removes most of the impulse noise.

Acknowledgment

The writer wishes to acknowledge the kind assistance of Mr. Robert Adler and Mr. C. W. Carnahan in this development.

Bibliography

(1) Heinrich Barkhausen, "Lehrbuch der Elektronenroehren," S. Hirzel, Leipzig, Germany, 1935 [©1929].
(2) Hikosaburo Ataka, "On superregeneration of an ultra-short-wave receiver," Proc. I.R.E., vol. 23, pp. 841–885; August, 1935.
(3) Frederick W. Frink, "The basic principles of superregenerative reception," Proc. I.R.E., vol. 26, pp. 76–107; January, 1938.
(4) Henry P. Kalmus, United States Patent No. 2,233,706.

Appendix

Assuming that the first live period starts with the time zero and that the phases of the natural and the controlling waves at that time are the same, $\phi(t)$ in (3) must increase by $2\pi(f_2-f_1)T$, where $T=1/F$, for each quench period. During the nth quench period after $t=0$, equation (3) becomes

$$W = A(t) \sin [2\pi f_1 t + 2\pi(f_2 - f_1)(n-1)T]. \quad (4)$$

The function $\phi(t)$ is the "staircase" function shown in Fig. 16. It is not periodic unless (f_2-f_1), and F are com-

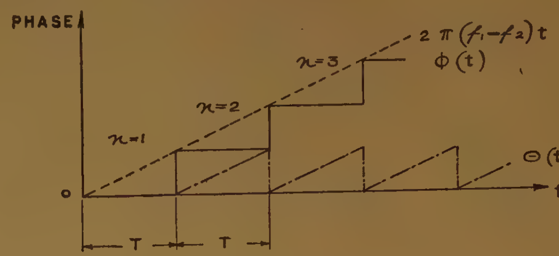


Fig. 16—The ϕ and θ functions plotted against time as described.

mensurable in which case $(f_2-f_1)(n-1)T$ assumes an integral value for some value of n, and the function repeats for further values of n.

However, we may rearrange (4) by adding and subtracting $2\pi f_2 t$ to the argument of the sine function, and obtain $W=A(t)\ \sin\ \{2\pi f_2 t+2\pi[f_1-f_2][t-(n-1)T]\}$. The last term in the argument of the sine function is a new function $\theta(t)=2\pi[f_1-f_2][t-(n-1)T]$ which is periodic with T, as we can see, if we combine the slope function represented by the term which is linear with t with the "staircase" function represented by the $(n-1)T$ term, thereby arriving at a saw-tooth function as shown in Figs. 16 and 10.

A Broadcast-Studio Control Console*

R. H. DeLany†, nonmember, I.R.E.

Summary—**This paper gives a brief description of a studio console which has some unique features. The design is esthetic in appearance, is made for the comfort and convenience of the operator, and last but not least the equipment has complete accessibility for maintenance and repair. The vertical mixers increase the ease, accuracy, and speed of operation making it possible to handle as many as six mixers simultaneously.**

Early in 1941, the United Broadcasting Company decided to modernize its studio control equipment. From five of its studios programs are fed to the Mutual Broadcasting System in addition to the continuous program service furnished to the WHK and WCLE transmitters. The old equipment had been in continuous service for over 10 years so it was time to modernize with new equipment.

Engineers, operators, and announcers co-operated in offering opinions and suggestions with regard to the facilities to be embodied in the new equipment necessary for present-day program-production requirements. After analyzing all the suggestions furnished, a general plan was evolved. Circuits were developed to incorporate the practical ideas. A plywood mockup was made of the proposed console cabinet. The studio operators gave their constructive criticisms of the physical dimensions. After a few alterations the mockup sample was ready to be used as a model for the fabrication of the consoles.

The framework and sides of the consoles are made of steel. To prevent wear and corrosion the footrest is made of stainless steel. The top of the table and console is made of masonite to give warmth to the touch. Each cabinet is finished differently by welding onto the console inlaid lacquer finishes. This, of course, was done arbitrarily but with pleasing color schemes. For example one combination consisted of green alligator table and console tops, reddish teakwood sides, and a green marble pedestal.

Fig. 1 shows the general appearance and location of all operating controls. First consideration was given to locating them for convenience and accessibility and second, to maintain as great a degree of symmetry as practicable. There is a seven-channel mixer and master volume control mounted on the panel directly in front of the operator. There are volume controls for six

* Decimal classification: R612.1. Original manuscript received by the Institute, February 24, 1943; revised manuscript received, June 26, 1944.
† Chief Engineer, WHK, United Broadcasting Company, Cleveland, Ohio.

microphones flanked on the left end by a line control and on the right end by the master gain control. A second line from master control can be patched into any one of the six studio mixer positions. This could, for example, be used to supply the studio with the second network program and thus make it possible to use the studio control room as an emergency master control room for one of the stations.

The VU meter and clock are self-illuminated and located in the center immediately below the operator's line of sight into the studio. To the right of the clock is a bank of seven keys mechanically interlocked for selecting monitor channels. Four of these keys are active with two spares as shown in the functional diagram of Fig. 2. To the left of the VU meter, the utility and monitor volume controls are mounted with room for a public-address volume control between. The larger studios require the sound reinforcement for an audience. A key located to the left of the VU meter should be used to connect this meter to the outgoing program line through jacks and, at the same time, light the lamps inside the meter case.

Just to the left of the mixer panel is a tube-checking and signaling panel. The condition of the tubes is checked by a multicontact selector switch which connects the meter across a small resistor in the cathode circuit of any tube used in the air channel. The two keys control signaling lamps on the master control console for WHK and WCLE, respectively.

To the right of the mixer panel there is the intercommunicating panel containing a microphone, speaker, and two keys. One of the keys permits the operator to speak into the studio when the audition monitor selector key has been operated. The other key allows the operator to talk to master control at any time. The master-control operator has similar facilities for communicating through the speaker in this panel to the studio-control operator. This intercommunication feature speeds up the operating procedure and frees the telephone facilities needed for other purposes.

All equipment is connected through double jacks allowing for complete flexibility. This makes it easy to substitute for faulty equipment. The low-level preamplifiers and the line amplifier are located on the right-hand side; and power supplies, communication, transcription cue, monitor, and public-address amplifiers are located on the left-hand side.

Particular attention was given to both the electrical and mechanical features of the mixer design. Electrically these mixers are bridged-T attenuators with a constant characteristic resistance of 600 ohms. They are provided with 30 steps of 1.5 decibels per step-up to near cutoff where the taper is rapid to approach infinite attenuation at cutoff. The only difference between the mixers and the master gain control is that the latter has a detent plate. The mixers and master gain control are matched to each of their respective 600-ohm lines by similar circuits from a common point. Hence, for example, if the master gain control becomes noisy or otherwise defective, the input to one of the other mixers can be patched to the input of the line amplifier to continue with the program. Above each mixer is an on-off key which connects a matching resistance into the mixer when the preamplifier is disconnected.

Fig. 1—Front view of console.

Mechanically the mixers are made for long life and hard wear. The contact studs are made of silver alloy to give long wear at a low noise level. They are completely enclosed to minimize the collection of dust. If a mixer should become noisy it can be readily cleaned from the front by removing the escutcheon plate which is held in position by two small thumbscrews.

Vertical mixers have been used in WHK's studios as standard equipment since 1930. They have proved to be far superior to the rotary type. A survey of all the operators, including those accustomed to rotary-type mixers, revealed a decided preference for the vertical mixer providing they have the equivalent electrical characteristics and mechanical sturdiness. These features have been incorporated in the present design. This type of mixer also gives a more easily seen picture of the gain-setting of the controls. This lever-type control provides positive and quick action when it is necessary to change or cut off a microphone in a hurry. With a little training an operator can manipulate three mixers with each hand.

The accessibility of all the equipment is seen in Fig. 3. The four large double-hinged doors allow for immediate access to all tubes and wiring. These doors are hinged in the middle with a snap-fastening feature that permits only half the door to open at first. If a larger opening is needed the second half of the door is opened or the complete door can be quickly unhinged and laid aside. All wiring connections are visible through the outer doors, while tube replacements are effected by way of the inner doors.

The top of the console is readily removable and the mixer panel is hinged so it can be tipped forward onto the shelf. These features give complete freedom for maintenance and repair. The clock can be set from the front of the panel.

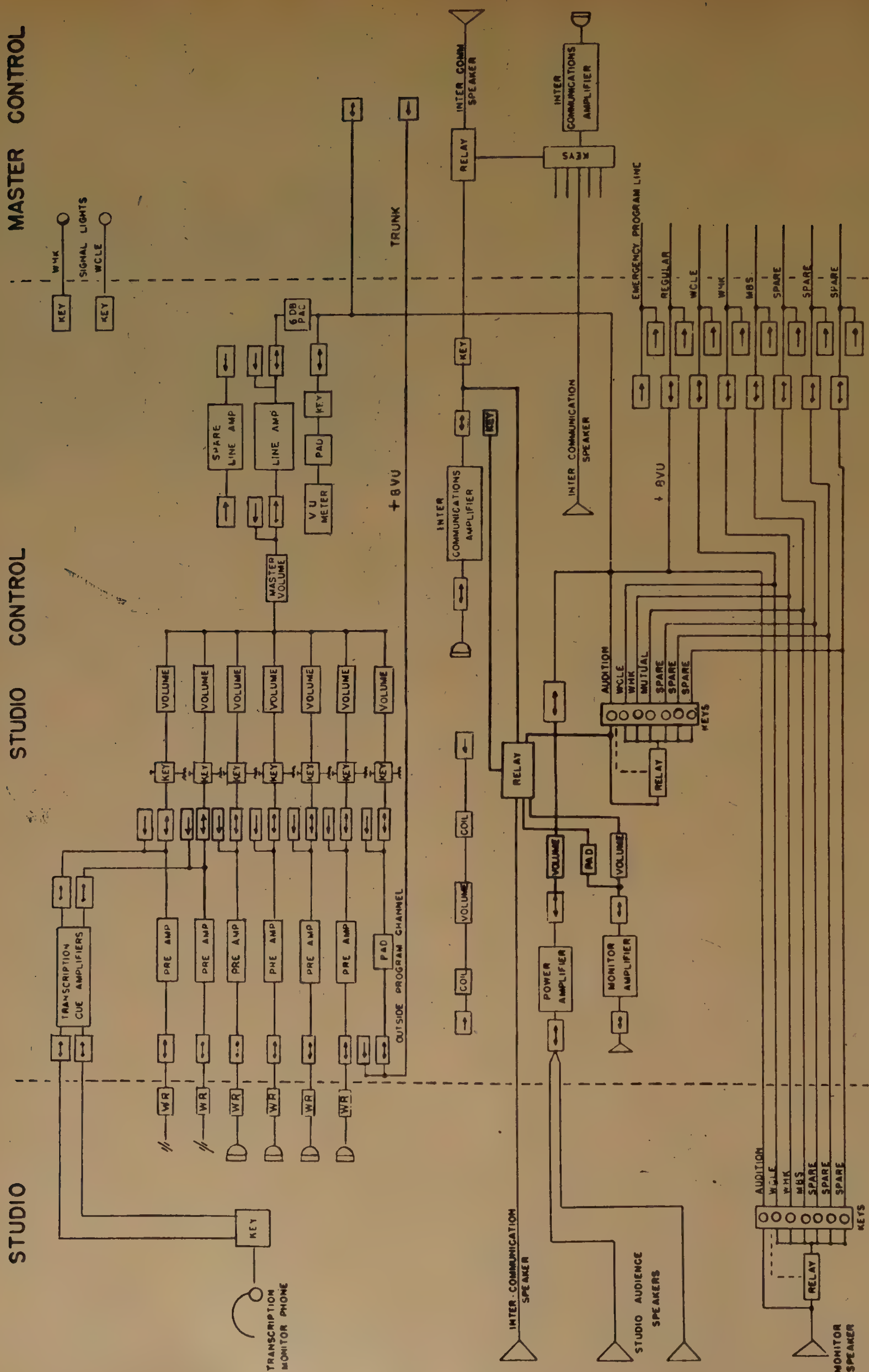

STUDIO
STUDIO CONTROL
MASTER CONTROL
TRANSCRIPTION CUE AMPLIFIERS
TRANSCRIPTION MONITOR PHONE
KEY
WR
PRE AMP
PAD
OUTSIDE PROGRAM CHANNEL
VOLUME
MASTER VOLUME
LINE AMP
SPARE LINE AMP
VU METER
6 DB PAD
+8VU
TRUNK
SIGNAL LIGHTS
W4K
WCLE
INTER COMMUNICATIONS AMPLIFIER
INTER COMM SPEAKER
RELAY
KEYS
COIL
INTER COMMUNICATION SPEAKER
EMERGENCY PROGRAM LINE
REGULAR
WCLE
W4K
WBS
SPARE
SPARE
SPARE
AUDITION
MUTUAL
POWER AMPLIFIER
MONITOR AMPLIFIER
INTER-COMMUNICATION SPEAKER
STUDIO AUDIENCE SPEAKERS
MONITOR SPEAKER

Fig. 2

Fig. 3 also shows the power switches and fuses behind the two doors on the left front surface of the console, while the supply of patch cords is kept behind the single door on the right-hand side. The upper door on the left-hand side gives access to 115-volt facilities. Here is found the master alternating-current power-line switch, air-channel power switch, auxiliary equipment power switches, and three convenience power outlets. When the lower door is opened access is given to the transfer key which permits the selection of one of two power supplies, either of which is capable of furnishing filament and plate voltages to the six preamplifiers and one program amplifier. The filament and plate circuits of all air-channel amplifiers have fuse protection here. Also, the main alternating-current line, the clock, and auxiliary switches are separately fused. A drawer is provided below the table for storing earphones, spare fuses, program schedules, etc.

The functional diagram of the studio facilities is shown in Fig. 2. The announcers, who run all recording and transcription equipment, are provided with an earphone monitor which can be switched to the output of any equipment. This enables the announcer to monitor recordings and cue the start of announcements to get a better production of such programs. A standard audio level of +8 VU is maintained in all circuits between master control and the studios to minimize crosstalk.

Most of the equipment used is standard produced by a nationally known manufacturer. The design of the console, construction of the auxiliary equipment, along with the assembly, wiring, and installation of the completed consoles was accomplished by members of the engineering department of the United Broadcasting Company.

It is hoped that some of the ideas presented in this article will be helpful to others confronted with similar problems.

Fig. 3—View of console with open access doors.

Acknowledgment

The writer wishes to acknowledge the individual and co-operative efforts of the United Broadcasting Company personnel who have made this project a success and especially to Carl E. Smith, development and design engineer, now on leave with the United States Army Signal Corps, who directed the project through the early stages.

Loop Antennas with Uniform Current*

DONALD FOSTER†, NONMEMBER, I.R.E.

***Summary*—The properties of a circular loop carrying uniform current are calculated for loops of any size relative to the wavelength. The radiation resistance and the greatest directivity pass through a series of maxima and minima as the frequency is increased. At frequencies below that for which one wavelength is contained in the circumference, the directivity graph is nearly independent of frequency. As the frequency is increased, additional lobes appear, the principal lobe tending to point more nearly in the direction normal to the loop. The paper includes a note on other loops, and a mathematical appendix dealing with certain integrals involving Bessel functions.**

Introduction

Loop antennas having substantially uniform current and dimensions comparable with or larger than the wavelength were first described in a paper which was presented by the author before a joint meeting of the I.R.E. and U.R.S.I. in April, 1937. In that paper, the method of driving such loops in segments so as to achieve uniformity of current was described, and expressions for the directivity and radiation resistance were derived. The original paper was submitted for publication in the PROCEEDINGS of the Institute of Radio Engineers in 1939. Publication has been delayed, due to previous inability of the author to satisfy editorial requests for compression of the analysis. As here presented, the results and scope of the paper are unchanged; but the mathematics is shorter. An application of these ideas to commercial purposes was described by Alford and Kandoian[1] in 1940.

There are various ways of obtaining a current distribution which is nearly uniform in magnitude and phase, even when the wavelength is small compared

* Decimal classification: R125.3. Original manuscript received by the Institute, April 17, 1939; revised manuscript received August 14, 1944.

† Stevens Institute of Technology, Hoboken, N. J. Now on leave of absence.

[1] A. Alford and A. G. Kandoian, "Ultra-high-frequency loop antennas," *Trans. A.I.E.E.* (*Elec. Eng.*, December, 1940), vol. 59, pp. 843–848; December, 1940.

with the distance around the loop. These methods depend on exciting the periphery in segments which are short in comparison with the wavelength, and arranging the lines which feed the segments so that, due to proximity of equal and opposite currents, the radial lines will not radiate. In one simple arrangement the loop consists of sectors of a circle or polygon driven in parallel by means of radial transmission lines.

Because of its simplicity and general resemblance to regular polygonal loops, the theory of the circle is given in detail.

Equivalent Current Moment of a Circular Loop

From the symmetry of the circle it is apparent that the field at any point, P, whose spherical polar co-ordinates are (r, θ, ϕ) is independent of the longitude. (See Fig. 1.) We may therefore, without any loss of generality, suppose that the line OP always lies in the plane $\phi=0$. Consider two equal infinitesimal segments of the

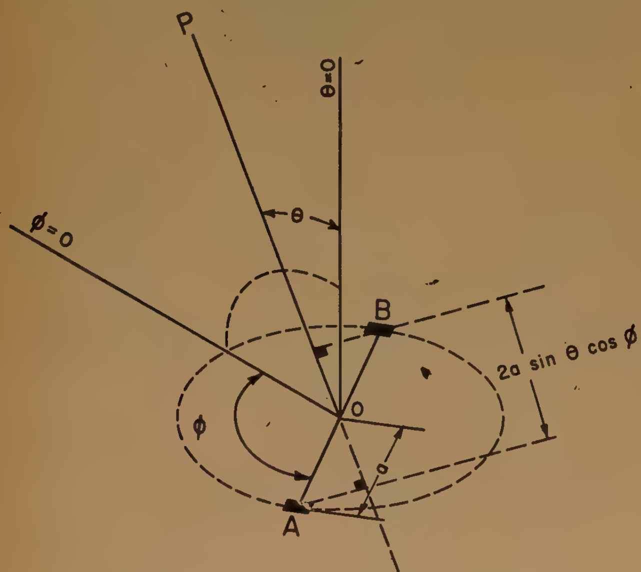


Fig. 1—Co-ordinate system for a circular loop.

circle, A and B, at the ends of a diameter. If we call the radius of the circle a, the co-ordinates of this pair of segments are $A(a, \pi/2, \phi)$ and $B(a, \pi/2, \phi+\pi)$. The moments of the currents of these elements, both measured in the direction of the tangent to the circle at A, are $Iad\phi$ and $Iae^{i\pi}d\phi$. The difference of phase of their signals at a remote point P is π plus 2π times the number of wavelengths in the projection of the diameter on the line OP. In symbols this difference of phase is $\epsilon=\pi+2\pi(2a \sin\theta\cos\phi)/\lambda$. We may replace the pair of elements with a single element the amplitude of whose moment is the sum of the amplitudes of the individual sources multiplied by $\cos(\epsilon/2)$ and whose phase is the average of the individual phases. Thus each pair of diametrically opposite elements may be replaced by one at the center, pointing in the $(\pi/2, \phi+\pi/2)$ direction, whose moment is

$$dF = 2iIa \sin(\beta a \sin\theta\cos\phi)d\phi; \quad (\beta \equiv 2\pi/\lambda).$$

This vector makes an angle ϕ with the normal to the plane, $\phi=0$. Hence, as observed at any remote point P the loop is equivalent to a directive point source at the center whose current moment is

$$F = F_\phi = \int_0^\pi 2iIa \sin(\beta a \sin\theta\cos\phi)\cdot\cos\phi\cdot d\phi$$
$$= 2\pi i I a J_1(\beta a \sin\theta)$$

where $J_1(\beta a \sin\theta)$ is the Bessel function of order unity.

The Field Intensities, Directivity, and Radiation Resistance of a Circular Loop

From this we can write down the following expressions for various quantities in the remote field: The magnetic vector potential is $A=A_\phi=e^{i\beta r}F/4\pi r$ which is in the direction of increasing ϕ. There is no θ component. The magnetic intensity,

$H=H_\theta=-\partial A_\phi/\partial r=i\beta A_\phi=-I(\beta a e^{-i\beta r}/2r)J_1(\beta a \sin\theta)$,

is wholly in the direction of increasing θ. For the electric intensity we have

$$E = E_\phi = -120\pi H_\theta = (e^{-i\beta r}/r)\cdot 60\pi I\beta a J_1(\beta a \sin\theta)$$

which is wholly in the ϕ direction. The average Poynting flux is

$$s = (15\pi/r^2\lambda^2)|F|^2$$
$$= (15\pi/r^2)(\beta a)^2 I_0^2 J_1^2(\beta a \sin\theta).$$

The power radiated per unit solid angle is

$$\Phi = r^2 s = 15\pi I_0^2(\beta a)^2 J_1^2(\beta a \sin\theta)$$

and the total power radiated is the integral of this with respect to solid angle for all directions—

$$P = \int \Phi d\Omega = \int_0^\pi \int_0^{2\pi} 15\pi I_0^2(\beta a)^2 J_1^2(\beta a \sin\theta)\cdot\sin\theta d\theta d\phi$$
$$= 30\pi^2 I_0^2(\beta a)^2 \int_0^\pi J_1^2(\beta a \sin\theta)\sin\theta d\theta$$
$$= 30\pi^2 I_0^2 \beta a \int_0^{2\beta a} J_2(y)dy.$$

The directivity is

$$d \equiv \frac{\Phi}{\Phi \text{ average}} = \frac{\Phi}{P/4\pi}$$
$$= 2J_1^2(\beta a \sin\theta)\cdot\left((1/\beta a)\int_0^{2\beta a} J_2(y)dy\right)^{-1}.$$

The radiation resistance is

$$R = \frac{P}{(1/2)I_0^2} = 60\pi^2\beta a \int_0^{2\beta a} J_2(y)dy.$$

The definite integral

$$f(x) \equiv \frac{1}{x}\int_0^{2x} J_2(y)dy; \qquad (x \equiv \beta a)$$

which appears in the expressions for power, directivity, and radiation resistance, is not a tabulated function; but it may be expressed in terms of tabulated functions. For values of x between $x=0$ and $x=5$ the integral $\int_0^{2x} J_0(t)dt$ is given at intervals of 0.01 in a table published

by Lowan and Abramowitz,[2] in 1943. The relation between $f(x)$ and the tabulated integral is

$$f(x) = \frac{1}{x}\int_0^{2x} J_2(y)dy = \frac{1}{x}\left[\int_0^{2x} J_0(y)dy - 2J_1(2x)\right].$$

The values of $J_1(2x)$ are available in many tables.[3] For values of x above the upper limit of the table of Lowan and Abramowitz the following asymptotic development is satisfactory:

$$f(x) \sim 1/x[1 - (1/\pi x)^{1/2}\{\sin(2x - \pi/4) + (11/16x)\cos(2x - \pi/4)\}].$$

At $x=5$, comparison with the tables shows a difference of 0.1 per cent, and the error diminishes as x increases.

For small values of x, the use of tables may be avoided by using the series

$$f(x) = (x^2/3)[1 - x^2/5 + x^4/56 - x^6/1080 + x^8/31680 - \cdots].$$

Four terms of this series result in an error of about 1.8 per cent at $x=2$; and one term is sufficient for this accuracy at $x=0.3$. The series is obtained by integrating the ascending series for the Bessel function of order 2.

Simple Approximations Valid at Low and High Frequencies

At low frequencies such that the length of the circle is a third of a wavelength or less, we have with good accuracy

$$f(\beta a) \doteq (\beta a)^2/3; \qquad J_1^2(\beta a \sin\theta) \doteq ((\beta a)^2 \sin^2\theta)/4$$

so that

$$d = (3/2)\sin^2\theta; \qquad R = 20\pi^2(\beta a)^4 \text{ ohms}; \qquad (\beta a < 1).$$

Approximation formulas valid at high frequencies may be obtained by using only the first term $(1/\beta a)$, of the asymptotic series for $f(\beta a)$. This is equivalent to saying that

$$\int_0^{2x} J_2(y)dy \doteq \int_0^{\infty} J_2(y)dy = 1.$$

The relative error in this approximation oscillates about the value *zero* as long as x is greater than 1.8. The amplitude of the oscillation diminishes slowly with increasing x. The resulting approximate expressions for d and R are

$$d = 2\beta a J_1^2(\beta a \sin\theta); \qquad R = 60\pi^2\beta a; \qquad (\beta a \geqq 5).$$

If $2\beta a$ happens to lie halfway between any adjacent roots of J_2 after the first, these expressions are nearly exact. The largest errors occur when $2\beta a$ is a root of $J_2=0$. These expressions show that, as the frequency is increased, d(max) increases without limit and that the relation between radiation resistance and frequency tends to become linear.

Graphs of Numerical Data. Circular Loop

These results are shown graphically in Figs. 2 to 6. Fig. 2 is the graph of $J_1^2(t)$. To use it as a universal sketch of relative directivity as a function of θ at any frequency, divide the axis of t between $t=0$ and $t=\beta a$ as a scale of θ from 0 to 90 degrees by means of the auxiliary quadrant as shown in the example, where $\beta a=6$.

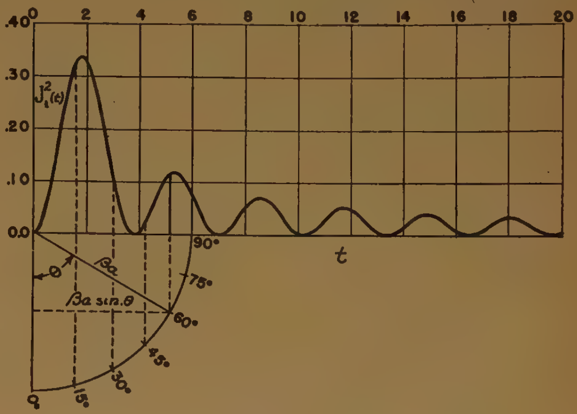


Fig. 2—Universal relative directivity chart.

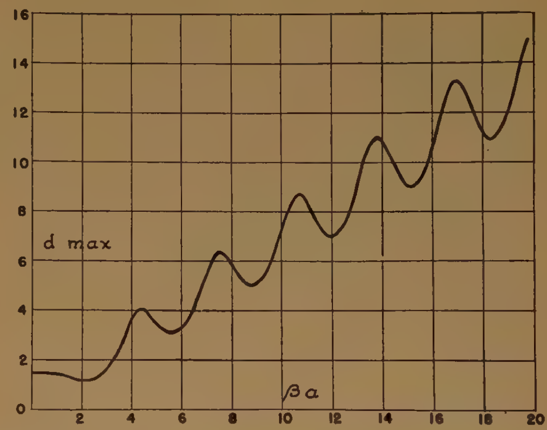


Fig. 3—Dependence of the greatest directivity on the number of waves in a circumference.

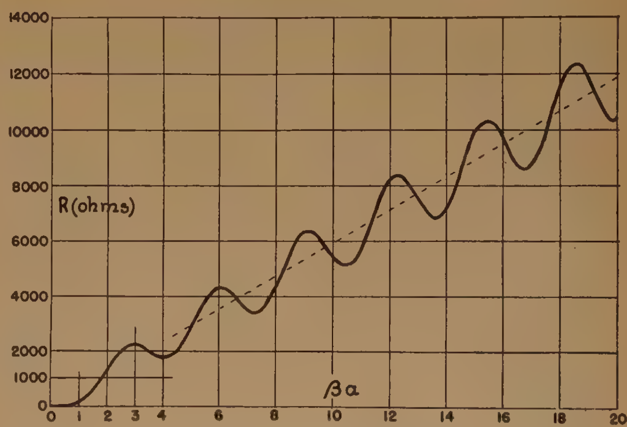


Fig. 4—Radiation resistance in ohms versus the number of waves in a circumference.

The maximum directivity, the directivity for the direction in which it is greatest for a given frequency, is shown as a function of the number of waves in a circumference by Fig. 3. For low frequencies such that $\beta a \leqq 1.84$ the maximum directivity is in the direction of the plane of the loop ($\theta = 90$ degrees). For higher frequencies the

[2] A. N. Lowan and M. Abramowitz, *Jour. Math. and Phys.*, vol. 22, pp. 2–12; May, 1943. This is available from the United States National Bureau of Standards as TM-20.

[3] British Association Mathematical Tables, vol. 6, part I, Cambridge University Press, Cambridge, England, 1937.

direction for maximum directivity is such as to make $\beta a \sin \theta = 1.84$.

The way in which radiation resistance depends on the number of waves in a circumference is shown by Fig. 4. The straight line in this figure represents the linear approximation referred to in the previous section.

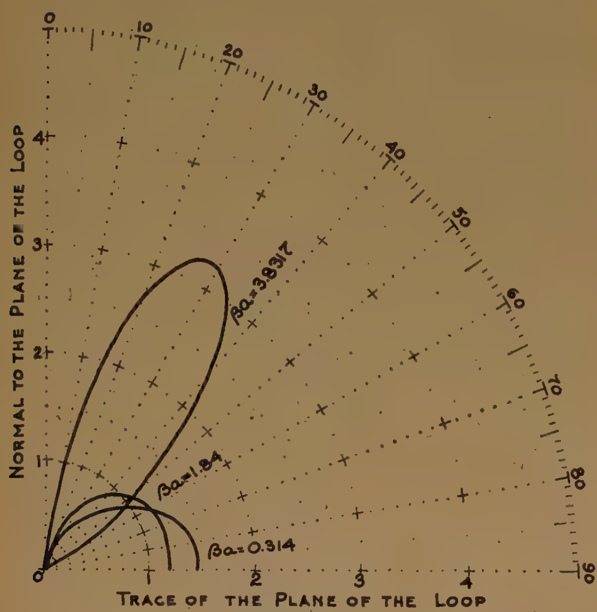


Fig. 5—Polar absolute-directivity graphs for $\beta a = 0.3142$, 1.841, 3.832.

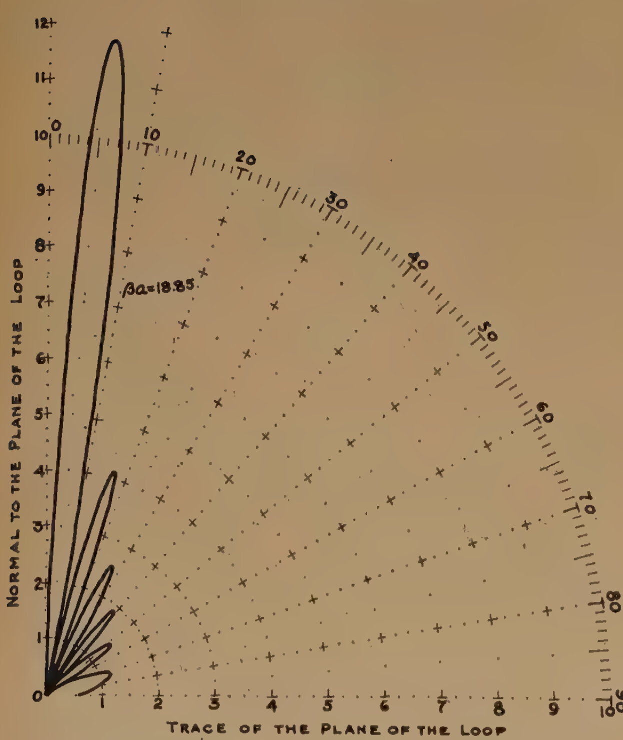


Fig. 6—Typical high-frequency absolute-directivity graph. ($\beta a = 6\pi$.)

Figs. 5 and 6 are typical polar-directivity curves for various frequencies. For frequencies below that for which the wavelength equals the circumference, the directivity graph is almost independent of frequency. The manner in which additional lobes appear at higher frequencies is seen most easily with the aid of Fig. 2.

Other Loops with Uniform Current

It is a simple matter to write general formulas for the θ and ϕ components of the equivalent current moment of a polygonal loop. As is well known, the shape has no effect on the properties of the antenna at low frequencies. At high frequencies the properties of regular polygons approach those of the circle when the number of sides is increased.

As an example of a polygonal loop having a small number of sides, let us consider the square. From symmetry it is clear that the electric vector is always in the plane of the loop and the whole variation of the directivity pattern with ϕ (azimuth with respect to a side) occurs in a range of $\pi/4$. In other words, lines joining opposite vertices and the middles of opposite sides are axes of symmetry of the directivity pattern. Comparing the directivity patterns of the square and the circle in a plane normal to the loop and parallel to a side, we find that while the variation of directivity with θ for the circle is expressed by the factor

$$J_1^2(\beta a \sin \theta); \qquad (a = \text{half the diameter})$$

for the square the corresponding factor is

$$\sin^2 (\beta a \sin \theta); \qquad (a = \text{half the side}).$$

The first-order Bessel function resembles the sine function in a general way, having about the same values of the argument for its zeros and extremes; but the loops of the θ directivity pattern of the square are all of the same length, whereas in the case of the circle the length of the loop diminishes with the order of interference.

Appendix

1. *Proof that* $\int_0^\pi \sin (k \cos x) \cos x dx = \pi J_1(k)$

Starting with Bessel's definition of $J_n(k)$, we have $J_n(k) = (1/2\pi)\int_0^{2\pi} \cos (n\theta - k \sin \theta) d\theta$; and in particular, $J_0(k) = (1/2\pi)\int_0^{2\pi} \cos (k \sin \theta) d\theta$. Changing the variable from θ to $(\pi/2 - x)$, and bisecting the range of integration gives $J_0(k) = (1/\pi)\int_0^\pi \cos (k \cos x) dx$. Differentiating with respect to k gives $J_0'(k) = (1/\pi)\int_0^\pi (\partial/\partial k) \cos (k \cos x) dx = -(1/\pi)\int_0^\pi \sin (k \cos x) \cdot \cos x dx$. Substituting $-J_0'(k) = J_1(k)$ we obtain

$$\int_0^\pi \sin (k \cos x) \cdot \cos x dx = \pi J_1(k)$$

2. *Evaluation of the integral* $\int_0^\pi J_n^2(\beta a \sin \theta) \sin \theta d\theta = f_n(\beta a)$

Using the series[4] for $J_n^2(z)$

$$J_n^2(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{2m+2n} \Gamma(2m+2n+1)}{m!\Gamma(m+2n+1)\{\Gamma(m+n+1)\}^2}$$

which is valid when $2n$ is not a negative integer, we have

$$f_n(x) \equiv \int_0^\pi J_n^2(x \sin \theta) \sin \theta d\theta$$

[4] G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge University Press, Cambridge, England, 1922, p. 147.

$$= \sum_{m=0}^{\infty} \frac{(-1)^m (x/2)^{2m+2n} \Gamma(2m+2n+1)}{m!\Gamma(m+2n+1)\{\Gamma(m+n+1)\}^2} \cdot \int_0^{\pi} (\sin\theta)^{2m+2n+1} d\theta.$$

The integral under the summation sign is of the form $\int_0^\pi \sin^m x dx$. Starting with the integral definition of the beta function and substituting $x = \sin^2 u$, we have

$$B(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$
$$= 2\int_0^{\pi/2} (\sin u)^{2a-1} (\cos u)^{2b-1} du.$$

Putting $2b-1=0$ and $2a-1=m$, we obtain

$$B\left(\frac{m}{2}+\frac{1}{2}, \frac{1}{2}\right) = 2\int_0^{\pi/2} \sin^m u du = \int_0^\pi \sin^m u du.$$

The relation between the beta and gamma functions is

$$B(a, b) = \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}; \therefore \int_0^\pi \sin^m u du = \frac{\pi^{1/2}\cdot\Gamma(m/2+1/2)}{\Gamma(m/2+1)}.$$

It follows that

$$f_n(x) = \frac{(-1)^m (x/2)^{2m+2n}\Gamma(2m+2n+1)}{m!\Gamma(m+2n+1)\{\Gamma(m+n+1)\}^2} \times \frac{\pi^{1/2}\Gamma(m+n+1)}{\Gamma(m+n+3/2)}$$
$$= \sum_{m=0}^{\infty} \frac{(-1)^m (x/2)^{2m+2n}\Gamma(2n+2n+1)\cdot\pi^{1/2}}{m!\Gamma(m+2n+1)\cdot\Gamma(m+n+1)\cdot\Gamma(m+n+3/2)}.$$

By the duplication formula of Legendre,

$$\Gamma(m+n+1)\cdot\Gamma(m+n+3/2) = (1/2)^{2m+2n+1}\cdot\pi^{1/2}\cdot\Gamma(2m+2n+2)$$

and by a characteristic property of the gamma function,

$$\Gamma(2m+2n+2) = (2m+2n+1)\cdot\Gamma(2m+2n+1).$$

Hence $f_n(x) = \sum_{m=0}^{\infty} \frac{(-)^m 2x^{2m+2n}}{m!(2m+2n+1)\cdot\Gamma(m+2n+1)}$.

Comparing this with the ascending series[5] for $J_{2n}(2x)$

$$J_{2n}(2x) = \sum_{m=0}^{\infty} \frac{(-)^m x^{2m+2n}}{m!\Gamma(m+2n+1)}$$

we see that

$$(d/dx)\{(1/2)xf(x)\} = J_{2n}(2x).$$

Integrating from zero to x gives

$$\int_0^x d\{tf_n(t)\} = xf_n(x) - 0\cdot f(0)$$
$$= xf_n(x) = \int_0^x 2J_{2n}(2t)dt.$$

Substituting $t = y/2$, we have

$$xf_n(x) = \int_0^{2x} J_{2n}(y)dy$$

[5] See page 40 of footnote reference 4.

or

$$f_n(x) = (1/x)\int_0^{2x} J_{2n}(y)dy.$$

In particular, $f_1(\beta a) = (1/\beta a)\int_0^{2\beta a} J_2(y)dy$.

3. *Computation of the integral* $xf(x) \equiv \int_0^{2x} J_2(y)dy$

For values of x less than 1 or 2, the series which we have found for $f(x)$ is better than any tables. It is the same ascending power series as would be obtained by termwise integration of J_2; viz.,

$$xf(x) = 2x^2 \sum_{m=0}^{\infty} \frac{(-x^2)^m}{m!(2m+3)(m+2)!}.$$

The series converges for all values of x, but it is too slow for large values of x.

By means of the recurrence formula

$$J_{n-1}(z) - 2J_n'(z) = J_{n+1}(z)$$

we obtain

$$xf(x) = \int_0^{2x} J_2(y)dy = \int_0^{2x} J_0(y)dy - 2J_1(2x)$$

which enables us to use the tables[2] of the integral of the Bessel function of order zero, which cover the range from $2x=0$ to $2x=10$.

The integral may easily be expressed in terms of several convergent or asymptotic series of Bessel functions. But, for computation, the most convenient expression is obtained by integrating the asymptotic series[6]

$$J_n(x) \sim (2/\pi x)^{1/2}\{P_n(x)\cdot\cos(x - n\pi/2 - \pi/4) - Q_n(x)\cdot\sin(x - n\pi/2 - \pi/4)\}$$

$$P_n(x) \equiv \left\{1 - \frac{(4n^2-1^2)(4n^2-3^2)}{2!(8x)^2} + \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)(4n^2-7^2)}{4!(8x)^4} - \cdots\right\}$$

$$Q_n(x) \equiv \left\{\frac{4n^2-1^2}{1!8x} - \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)}{3!8x^3} + \cdots\right\}.$$

Integrating by parts, we obtain

$$\int_0^u J_n(x)dx = \int_0^\infty J_n(x)dx + \int_\infty^u J_n(x)dx$$
$$\sim 1 + (2/\pi u)^{1/2}\{\sin(u - n\pi/2 - \pi/4) + ((4n^2-5)/8u)\cos(u - n\pi/2 - \pi/4)\} + \text{etc.}^7$$

Hence

$$xf(x) = \int_0^{2x} J_2(y)dy \sim 1 - (1/\pi x)^{1/2}\{\sin(2x - \pi/4) + (11/16x)\cos(2x - \pi/4)\} + \cdots.$$

[6] See page 195 of footnote reference 4.

[7] Compare N. W. McLachlan, "Bessel Functions for Engineers," Oxford University Press, Oxford, England, 1941, p. xi. (The equation given in this book is not correct due to a mistake in sign.)

Improved High-Frequency Compensation for Wide-Band Amplifiers*

ALEXANDER B. BERESKIN†, ASSOCIATE, I.R.E.

Summary—In order to obtain the best results from wide-band amplifiers of the type employed in television video amplifier and cathode-ray amplifier circuits, it is usually desirable to provide a means for producing high-frequency-response compensation.

It is the purpose of this paper to show how, by proper design and at no additional cost, a single compensating element can be made to provide compensation equivalent to that obtainable in a circuit using two compensating elements and to discuss the characteristics of this type of compensation. The circuit under consideration will provide uniform response to a frequency approximately half an octave higher than that obtained with the single compensating element circuit extensively used at the present time.

AMPLIFIER circuits of the type shown in Fig. 1 (a) are extensively used when it is desired to obtain a uniform voltage amplification over a wide range of frequencies. It is well known that the voltage amplification at medium and high frequencies, for such a circuit, may be expressed as $A=g_mZ_L$ where Z_L is the magnitude of the equivalent load impedance in the plate circuit of the tube. At medium frequencies Z_L is for all practical purposes equal to R_L, the reactance of C_s being so high that it may be neglected completely. The plate resistance of the tube and R_g are usually so large compared to R_L that they can be neglected at all frequencies. At the higher frequencies, however, when the reactance of C_s becomes of the same order of magnitude as R_L, the value of Z_L will decrease with increasing frequencies and, therefore, the voltage amplification will decrease in the same manner as the ratio Z_L/R_L shown by the curve a of Fig. 2. In this figure f_0 is the value of frequency at which Z_L becomes $1/\sqrt{2}$ of its value in the middle-frequency range. The phase shift for this case is shown in curve a' of the same figure.

In the normal compensating circuit, Fig. 1 (b), an inductance having a value of $L=R_L/2\omega_0$ is used in series with R_L. The equivalent impedance of this combination and the corresponding voltage amplification tends to rise slightly for frequencies below f_0 and to drop rapidly for frequencies above f_0, being equal to the middle-frequency value at $f=f_0$. These relations are shown on curve b and the corresponding phase shifts on curve b' of Fig. 2. Higher values of L will produce overcompensation while lower values will produce undercompensation.

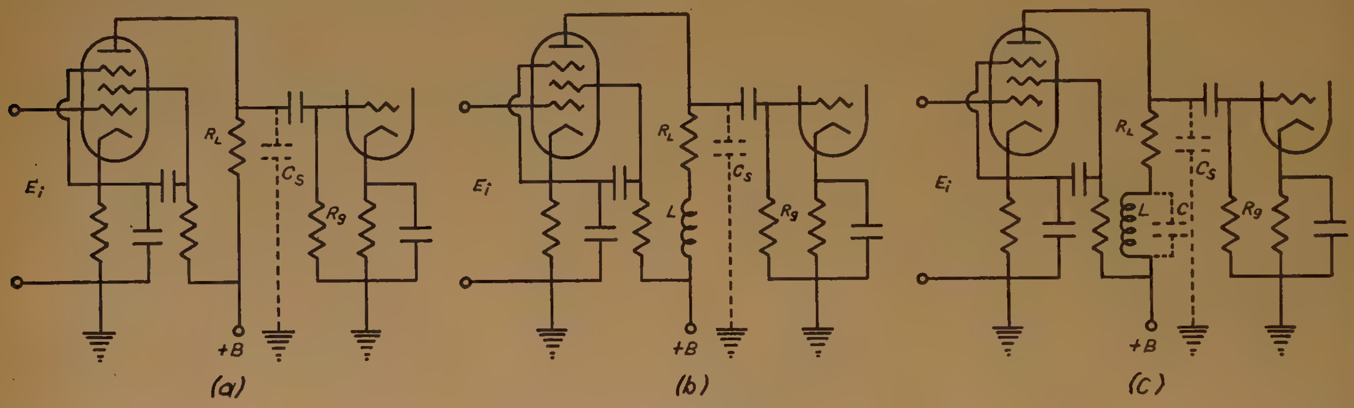


Fig. 1—Amplifier circuits.

Improved Circuit

It has been shown previously by Wheeler,[1] dealing with the subject from the filter-circuit-theory point of view, that by the use of additional compensating elements the range of uniform frequency response can be extended to an ideal maximum of $f=2f_0$. The compensation represented by the circuit of Fig. 1 (b), however, is the only one that has received appreciable popular approval in the simpler applications probably because of the additional cost and the more involved calculations required in the multielement circuits.

The particular circuit that this paper is concerned with is the one indicated in Fig. 1 (c) and it can be seen that the additional circuit element C is required. It will be shown, however, that in practice this element can be dispensed with if the inductance L is properly designed.

The analysis that follows was developed by the author independently of the Wheeler article and is presented on the basis of ordinary circuit theory which in this case is believed to yield the required information on a simpler basis than that obtainable with filter circuit theory.

Examination of Fig. 2 a and b shows that when

* Decimal classification: R363.1. Original manuscript received by the Institute February 28, 1944; revised manuscript received, June 17, 1944.
† University of Cincinnati, Cincinnati, Ohio.

[1] Harold A. Wheeler, "Wide-band amplifiers for television," Proc. I.R.E., vol. 27; pp. 429–437; July, 1939.

no inductance L is used the impedance Z_L decreases as f approaches f_0, while for $L=R_L/2\omega_0$ the impedance Z_L will rise slightly for values of f/f_0 less than 1.0. It is probable that some smaller value of L would tend to keep Z_L constant in this region and the exact value required is determined in the following manner. (See Fig. 1 (c)): Let ω_0 be defined by $R_L=1/\omega_0 C_s$ then $C_s=1/\omega_0 R_L$. Assume that the frequency is sufficiently low so that the effect of C can be completely neglected, yielding the circuit of Fig. 1 (b) from which:

$$\overline{Z}_L = \frac{(R_L + j\omega L)(-j1/\omega C_s)}{R_L + j(\omega L - 1/\omega C_s)} = \frac{L/C_s - jR_L/\omega C_s}{R_L + j(\omega L - 1/\omega C_s)}$$
$$= \frac{\omega_0 L R_L - j\omega_0 R_L{}^2/\omega}{R_L + j(\omega L - \omega_0 R_L/\omega)}.$$

Let $\omega_0 L = R_L/n$ then $L=R_L/n\omega_0$ so that

$$\overline{Z}_L = \frac{R_L{}^2/n - j(\omega_0/\omega)R_L{}^2}{R_L + j((\omega/\omega_0)R_L/n - (\omega_0/\omega)R_L)} = \frac{R_L{}^2(1/n - j(\omega_0/\omega))}{R_L + j((\omega/\omega_0)R_L/n - (\omega_0/\omega)R_L)}$$
$$= R_L\sqrt{\frac{(1/n)^2 + (\omega_0/\omega)^2}{1 + (\omega/\omega_0)^2(1/n)^2 - 2/n + (\omega_0/\omega)^2}}\,\Big|\tan^{-1}\omega_0 n/\omega - \tan^{-1}(\omega/\omega_0 n - \omega_0/\omega).$$

If the magnitude of $\overline{Z}_L$ is to remain constant then

$$(1/n)^2 + (\omega_0/\omega)^2 = 1 + (\omega/\omega_0)^2(1/n)^2 - 2/n + (\omega_0/\omega)^2$$
$$(1/n)^2[1 - (\omega/\omega_0)^2] + 2/n - 1 = 0$$

and assuming that the frequencies are such that $(\omega/\omega_0)^2$ is much less than 1.0, then $n^2-2n-1=0$ so that $n=2.415$. Therefore, to keep Z_L from rising above its medium-frequency value at frequencies below f_0, the required value of compensating inductance is $L=R_L/2.415\omega_0$.

A mathematical analysis does not conveniently yield the required information for values of f close to and above f_0, but this information can be obtained by investigating the effect of variations of f_R/f_0 (where f_R is the parallel resonant frequency of the LC combination of Fig. 1 (c)) on the family of response characteristics. A set of these curves was computed for the circuit of Fig. 1 (c) using the value of $L=R_L/2.415\omega_0$ as determined above. The results of this investigation are shown in the curves of Fig. 3. An especially interesting characteristic of these curves is the tenacious manner in which they tend to cling to the value of $Z_L/R_L=1.0$ for values of f/f_0 less than 1.0. It can also be seen that a variety of characteristics can be obtained by varying the value of C used thus varying f_R/f_0. The curves for $f_R/f_0=2.1$ were taken from this set of curves and transposed to Fig. 2 as curves c and c' where they indicate very clearly the improved response obtained. This improved response is obtained without any appreciable deterioration of the phase shift characteristic.

The curves of Fig. 3 were computed on the assumption that no losses existed in either the coil L or the condenser C and it is naturally desirable to determine what effect these losses would have on the practical application. For this reason the curves of Fig. 4 were calculated for the $f_R/f_0=2.1$ case for several values of Q_0, as indicated on the graph, where Q_0 is the quality of the inductance at the frequency f_0 and the condenser is assumed to have no losses. It is obvious from these curves that a value of Q_0 as low as 41.4 produces only a slight deviation in the curves over that obtained for $Q_0=\infty$. The curve for $Q_0=4.14$, while showing a deviation from the ideal curve, is still surprisingly close to it.

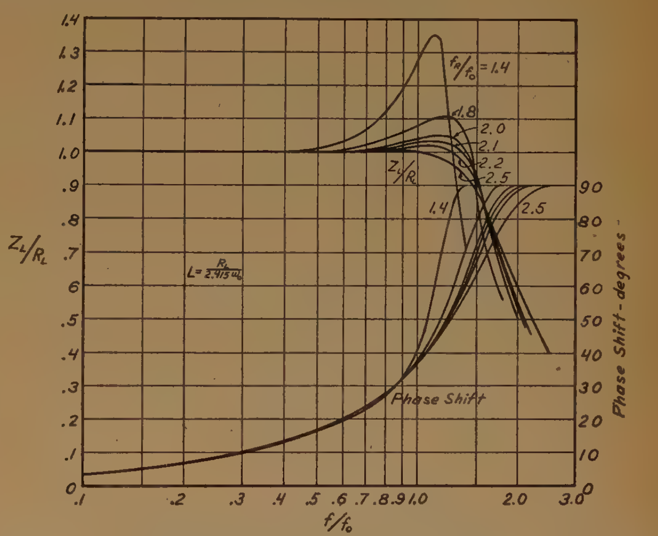


Fig. 3—Effect of f_r/f_0 ratio on response characteristics for "improved-compensation" circuit (theoretical).

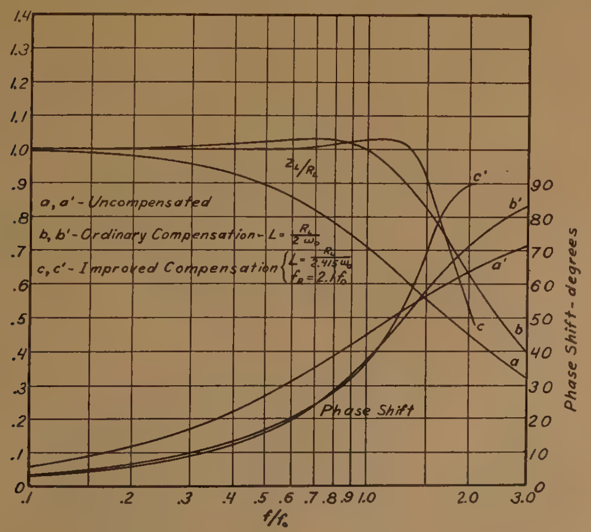


Fig. 2—Response characteristics for various amplifier circuits (theoretical).

All of the previous discussion has related to the theoretical determination of the response characteristics and it is naturally desirable to test these theoretical conclusions experimentally. With this view in mind, an experimental wide-band amplifier, using the circuit diagram of Fig. 1 (a) was set up and the response characteristic shown in Fig. 5 *a* was obtained. From this characteristic it was computed that 196 microhenries

were required, in the circuit of Fig. 1 (b), for the ordinary type of compensation. The response characteristic shown in Fig. 5 *b* was obtained but it did not check the

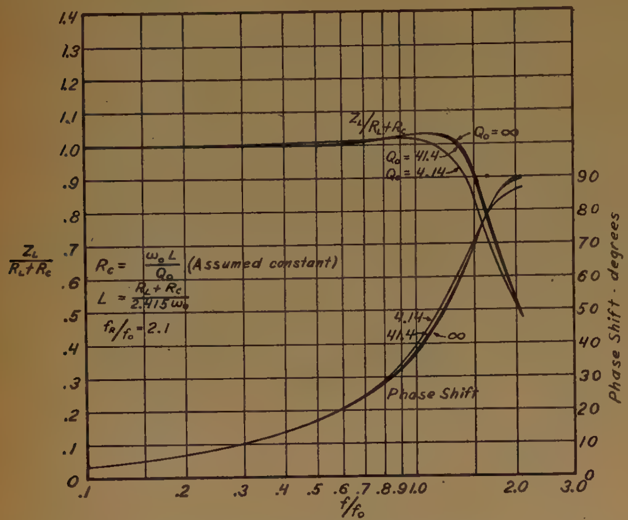


Fig. 4—Effect of coil Q on response characteristics for "improved-compensation" circuit (theoretical).

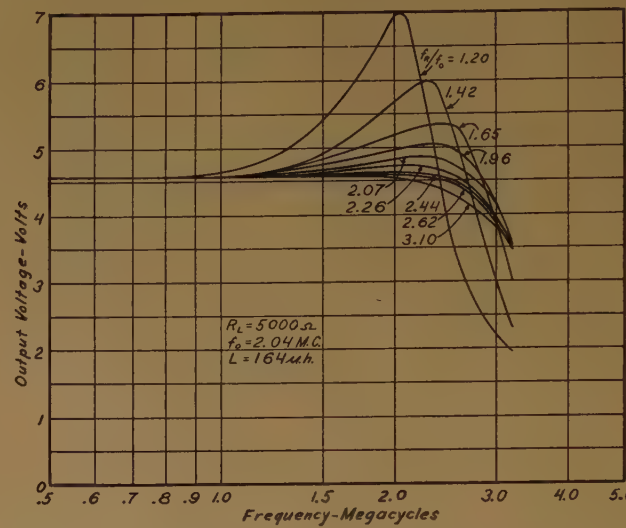


Fig. 6—Effect of f_r/f_0 ratio on response characteristics for "improved-compensation" circuit (experimental).

theoretical curve of Fig. 5 (c) for this type of compensation. Further investigation of the coil used, which was of the accepted universal-winding type, showed that it had a self-resonant frequency of 6.4 megacycles when measured by itself outside of the circuit. It is reasonable to assume that the additional lead capacitance, when connected in the circuit, would tend to lower this frequency slightly. A response curve for this same value of inductance, assuming a self-resonant frequency of 5.5 megacycles, was computed and is shown in Fig. 5 *d*. The agreement between this curve and the experimentally determined curve is quite close and explains

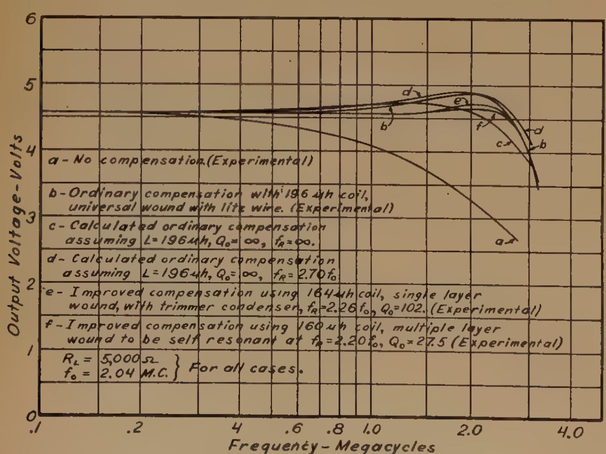


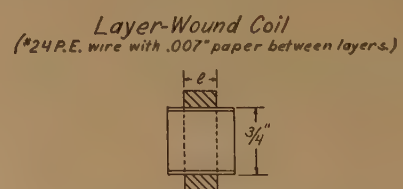
Fig. 5—Experimental and theoretical response characteristics for various circuits.

why in many cases the properly calculated ordinary compensation yields what appears to be a highly overcompensated response characteristic.

The use of a single-layer coil, of approximately the proper inductance ($L=164$ microhenries) for the improved compensation, shunted with a trimmer condenser which could be adjusted to give various values of f_R, yielded the experimental curves shown in Fig. 6, corroborating the theoretical curves of Fig. 3. The curve for $f_R/f_0=2.26$ was transposed to Fig. 5 as curve *e* and shows the manner in which the response is kept from overshooting at the high-frequency end.

A consideration of the values of C required in parallel with L and the corresponding values of f_R shows that these values are definitely within the range of self-resonant effects obtainable with coils, the only question being the manner in which these quantities can be properly controlled.

In the subsequent investigations, it was found that the self-resonant frequency of the coils could be very closely controlled by using layer-wound coils, thus re-



e	N	L	f_R
inches	turns	μ.h.	m.c.
9/32	77	164	3.45
1/4	74	163	3.75
7/32	77	163	4.08
3/16	78	163	4.60

Fig. 7—Effect of coil length on self-resonant frequency of a layer-wound coil (experimental).

versing the tendency of the past two decades toward the universal- or bank-wound coils. In these layers wound coils, the number of turns required for any given inductance, when the coils are wound on a form of a given diameter, changes only slightly with small changes

of length and height of the coil. If these turns form a short high coil, then there will be a relatively low distributed capacitance or a high self-resonant frequency. On the other hand, if the turns form a long low coil, of at least two layers, then the distributed capacitance will be high and the corresponding self-resonant frequency will be low. In between the two extremes almost any required value of distributed capacitance can be obtained.

A coil wound in the above manner had approximately the required inductance ($L=160$ microhenries) for the improved compensation and a value of $f_R/f_0=2.20$ with $Q_0=27.5$. The experimental response curve obtained with this coil is shown as curve f of Fig. 5. The agreement between this curve and curve e of the same figure, which was obtained with an actual condenser, is remarkably close.

The table in Fig. 7 shows the experimental variation of f_R for various coils having approximately the same inductance but different values of length and height. All of these coils were wound with No. 24 plain enameled wire with a layer of 0.007-inch insulating paper between each layer of wire.

Conclusions

The foregoing article has shown why ordinary compensation calculations very often produce what appears to be unexplained overcompensation. This is usually attributed to too high a value of inductance but may actually be due to the distributed capacitance of the coil tending to produce self-resonance at a frequency approximately two times the maximum frequency required from the amplifier.

The article has also shown the characteristics of, and the manner in which, the improved compensation associated with a two-element compensating circuit can be obtained, with the use of only one element. The one element is no more difficult to make, nor costs any more, than the original element used. A value of $L=R_L/2.415\omega_0$ is required and a value of f_R/f_0 of approximately 2.1 is indicated, the actual value used depending upon the response characteristic required.

In view of the additional benefits to be derived at no additional cost it appears reasonable that the method indicated here should be adopted in the future in all applications in which the ordinary type of compensation was previously used.

Antenna Design for Field-Strength Gain*

H. W. KOHLER†, ASSOCIATE, I.R.E.

Summary—An analysis is made of obtainable root-mean-square field strength in the horizontal plane with a given radio-frequency power fed into four identical, short vertical linear antennas which are located in the corners of a square. This field strength is compared to that produced with the same power fed into a single antenna of the same design. The resistance coupled into each antenna by the other three is computed.

A formula giving the gain in field strength for four antennas over that obtained from one antenna is derived, and families of gain curves are plotted.

The theory presented is in essential agreement with measurements of gain in field strength made at 200 and 400 kilocycles at the Pittsburgh, Pennsylvania, radio range. A gain in field strength of the order of 1.8/1 can be realized at the lower frequencies. The gain increases as the coil and ground resistance of the antenna circuit becomes greater, and it also increases as the antenna spacing is reduced.

At very small antenna spacings where the coupled reactance becomes comparable to the self-reactance of a single antenna, the field-strength gain decreases from the maximum value obtainable since the larger antenna tuning coils required in this case increase the losses.

Introduction

EXPERIMENTS conducted at the Pittsburgh radio range on 200 and 400 kilocycles have shown that a considerable gain in field strength can be obtained if a given radio-frequency power is fed into four antennas in phase instead of into a single antenna. The antennas are insulated towers 125 feet high, without top loading, located at the corners of a square whose diagonals measure 600 feet. For range operation the diagonal towers are excited 180 degrees out of phase with equal currents; whereas, for voice operation, where an essentially circular horizontal field pattern is desired, the antennas are fed in phase with equal currents. This is the method of operation which was used before the simultaneous radio range was developed.

The purpose of this paper is to check the available measurements theoretically, thereby obtaining relations between the several variables occurring in the problem which enable one to predict quantitatively the gain obtainable with similar four-element antenna arrays.

The procedure followed in the analysis is to find the root-mean-square value of the horizontal field pattern for four antennas with unit current in each antenna, and to determine the antenna current obtainable with a given radio-frequency power. This involves calculation of the resistance coupled into each antenna by the other three antennas, besides the radiation resistance of the reference antenna.

Expressions for the calculation of coupled resistance and reactance for linear vertical radiators are given in the literature.[1] In view of the short electrical length of the antennas involved here approximate values of

* Decimal classification: R320×R270. Original manuscript received by the Institute, February 18, 1944.

† Civil Aeronautics Administration, Department of Commerce, Washington, D. C.

[1] G. H. Brown and R. King, "High-frequency models in antenna investigations," Proc. I. R. E., vol. 22, pp. 457–480; April, 1934.

coupled resistance can be obtained by multiplying the values for quarter-wave antennas by the square of the ratio of the effective heights. Sinusoidal current distribution in the antennas is assumed, and the earth's surface is taken as a perfectly conducting plane.

The field strength in the horizontal plane obtained with four antennas is compared to that produced by a single antenna of identical height into which the same total power is fed, and the ratio of the two field strengths is plotted versus one half the diagonal spacing S, and versus the ratio of coil and ground resistance to radiation resistance (R_L/R_r), respectively.

Root-Mean-Square Field Strength of Horizontal Field Pattern of Four Antennas

Calling the diagonal spacing (in angular measure) of the antennas $2S$ and the length of the sides of the square $S\sqrt{2}$ (Fig. 1), the equation for the relative field strength

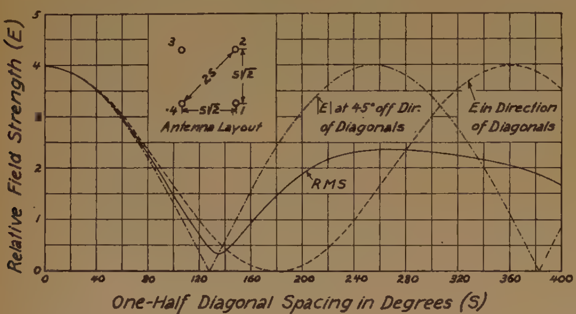


Fig. 1—Horizontal field strength versus antenna spacing. Antenna currents equal and in phase. Root-mean-square value of field strength taken in all directions of horizontal plane.

in the horizontal plane with in-phase unit current in each antenna is

$$E = 2[\cos (S \cos \theta) + \cos (S \sin \theta)] \qquad (1)$$

where θ is the azimuth angle measured from the diagonals.

The root-mean-square field strength in the horizontal plane is defined as

$$E_{\mathrm{rms}} = \left[1/(2\pi) \int_{-\pi}^{\pi} E^2 d\theta \right]^{1/2}. \qquad (2)$$

Introducing (1) into (2) and integrating gives

$$E_{\mathrm{rms}} = 2[1 + J_0(2S) + 2J_0(S\sqrt{2})]^{1/2} \qquad (3)$$

where J_0 designates the Bessel function of the first kind, of order zero. It will be observed that for very close spacing $J_0(2S) \doteq J_0(S\sqrt{2}) \doteq 1$, giving $E_{\mathrm{rms}} = 4$. This, of course, is the same result as obtained if we let $S = 0$ in (1), and if 4 units of current are fed into a single antenna. As S is increased the pattern becomes constricted, and for $S = 180$ degrees$/\sqrt{2} = 127.2$ degrees, four directions of zero field strength are obtained at 45 degrees off the diagonals. Field strengths in the direction of the diagonals and the bisectors, and root-mean-square field strengths, are shown in Fig. 1, plotted as a function of S.

Impedance of Each of Four Parallel Vertical Dipoles

The impedance Z_1 of a half-wave dipole in the presence of three other energized dipoles, all four carrying equal currents in-phase, is

$$Z_1 = Z_{11} + 2Z_{12} + Z_{13} = R_1 + jX_1 \qquad (4)$$

where Z_{11} = self-impedance of one dipole with the other dipoles removed.
Z_{12} = mutual impedance between adjacent dipoles.
Z_{13} = mutual impedance between diagonally opposite dipoles.

The derivation of the self- and mutual-impedance terms in (4) have been given.[2] For convenience the equations for Z_{11}, Z_{12}, and Z_{13} are reproduced below.

$$Z_{11} = 30[\gamma + \log_e (2\pi) - \mathrm{Ci}\,(2\pi) + j\,\mathrm{Si}\,(2\pi)] \text{ ohms} \qquad (5)$$

$$\gamma = 0.577 = \text{Euler's constant}$$

$$Z_{11} = 73.2 + j42.5 \text{ ohms}$$

$$\underset{\text{(ohms)}}{Z_{12} = 30} \begin{bmatrix} 2\,\mathrm{Ci}\,\beta S\sqrt{2} & -j2\,\mathrm{Si}\,\beta S\sqrt{2} \\ -\mathrm{Ci}\,\beta(\sqrt{2S^2+\lambda^2/4}+\lambda/2) & +j\,\mathrm{Si}\,\beta(\sqrt{2S^2+\lambda^2/4}+\lambda/2) \\ -\mathrm{Ci}\,\beta(\sqrt{2S^2+\lambda^2/4}-\lambda/2) & +j\,\mathrm{Si}\,\beta(\sqrt{2S^2+\lambda^2/4}-\lambda/2) \end{bmatrix} \qquad (6)$$

$$\underset{\text{(ohms)}}{Z_{13} = 30} \begin{bmatrix} 2\,\mathrm{Ci}\,\beta 2S & -j2\,\mathrm{Si}\,\beta 2S \\ -\mathrm{Ci}\,\beta(\sqrt{4S^2+\lambda^2/4}+\lambda/2) & +j\,\mathrm{Si}\,\beta(\sqrt{4S^2+\lambda^2/4}+\lambda/2) \\ -\mathrm{Ci}\,\beta(\sqrt{4S^2+\lambda^2/4}-\lambda/2) & +j\,\mathrm{Si}\,\beta(\sqrt{4S^2+\lambda^2/4}-\lambda/2) \end{bmatrix} \qquad (7)$$

$$\beta = 2\pi/\lambda$$

$$\mathrm{Ci}\,(u) = \int_{\infty}^{u} \frac{\cos x}{x}\,dx = \text{cosine integral} \quad \text{and} \quad \mathrm{Si}\,(u) = \int_{0}^{u} \frac{\sin x}{x}\,dx \doteq \text{sine integral.}$$

[2] P. S. Carter, "Circuit relations in radiating systems and applications to antenna problems," Proc. I.R.E., vol. 20, pp. 1004–1041; June, 1932.

Self-resistance and coupled resistance for quarter-wave antennas over a perfectly conducting plane are one half of the corresponding resistances of half-wave antennas in free space. Referring to Fig. 2, resistance and reactance obtained when looking into a quarter-wave radiator at the current maximum in the presence of three other energized quarter-wave radiators are shown as a function of the antenna spacing computed by (4) to (7), inclusive.

As the antennas are assumed to be perfect conductors, this input resistance ($1/2\ R_1$) is equal to the radiation resistance of each antenna. The relation of R_1 to the self-resistance and coupled resistance of each antenna is $R_1 = R_{11} + 2R_{12} + R_{13}$.

where R_{11} = radiation resistance of a half-wave dipole

R_{12} = coupled resistance between adjacent dipoles

R_{13} = coupled resistance between diagonal dipoles

Similarly we may write for the reactance terms $X_1 = X_{11} + 2X_{12} + X_{13}$.

For comparison, input resistance and reactance of a single quarter-wave radiator with the other radiators removed are also shown in Fig. 2, viz.,

$$1/2R_{11} = 36.6 \text{ ohms} \qquad 1/2X_{11} = 21.25 \text{ ohms}.$$

It will be observed that for small spacings the coupled resistance is positive and increases the radiation resistance. For $S=0$, $R_1 = 4\ R_{11}$ and $X_1 = 4\ X_{11}$.

For $S = 98$, 231, and 390 degrees the coupled resistance is zero, giving $R_1 = R_{11}$. Between 98 and 231 degrees the coupled resistance is negative. The minimum total resistance is found at $S = 150$ degrees and is 6.3 ohms.

Application to Radiators Shorter than One-Quarter Wavelength

The radiation resistance R_r of a thin vertical wire of effective height h whose lower end is near the surface of a perfectly conducting horizontal plane is given by the equations

$$\begin{aligned} R_r &= 1579(h/\lambda)^2 = 40 \tan^2 (\pi H/\lambda) \\ &= 10(\beta H)^2 \text{ ohms.} \end{aligned} \tag{8}$$

This expression is accurate for antennas much shorter than one-quarter wavelength. For antennas one-quarter wave long the above formulas give a value of radiation resistance 10 per cent too high. The effective height h is

$$h = \lambda/(2\pi) \tan (\pi H/\lambda) \tag{9}$$

where H = physical height of antenna.

For simplicity the coupled resistance of two identical short antennas is assumed to vary as the square of the effective height of the antennas based on the following reasoning:

$$\begin{aligned} E_2 &\propto I_1 h \\ V_2 &= hE_2 \propto h^2 I_1 \\ Z_{12} &= V_2/I_1 \propto h^2 \end{aligned}$$

where I_1 = root-mean-square value of maximum current in antenna 1

E_2 = axial field strength at radiator 2 due to current in radiator 1.

V_2 = total induced voltage in antenna 2.

While it is realized that the values of coupled resistance computed on this basis are not accurate, it is believed that they hold within a few per cent when applied to radio ranges.

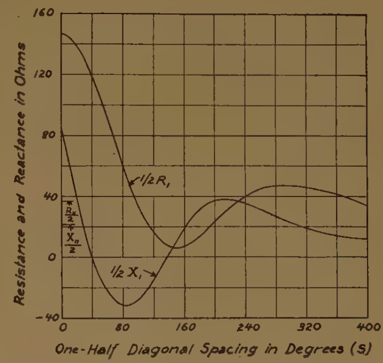


Fig. 2—Input resistance ($1/2\ R_1$) and reactance ($1/2\ X_1$) of each of four quarter-wave antennas above a perfectly conducting plane carrying equal currents in phase versus S. $1/2\ R_{11}$ and $1/2\ X_{11}$ are resistance and reactance of a single quarter-wave antenna.

In order to check the above procedure of calculating coupled resistance for short antennas, the following approximate formula was derived for the antenna layout in Fig. 1: (see Appendix)

$$R_c/R_r = 2\left[J_0^2(S/\sqrt{2}) - J_1^2(S/\sqrt{2})\right] + J_0^2(S) - J_1^2(S) \tag{10}$$

where J_0, J_1 = Bessel functions of first kind of order 0 and 1

R_c = resistance coupled into each antenna by the other three

R_r = radiation resistance of a single antenna with the others removed.

The equation is based on four vertical doublet antennas located in the corners of a square over a perfectly conducting horizontal plane. The antennas are assumed to have equal currents in phase, the currents increasing linearly from zero at the top to the maximum value at the ground plane, and the vertical field pattern of each antenna is assumed to vary with the cosine of the angle of elevation.

Table I gives a comparison of values of R_c/R_r calculated by cosine integrals and Bessel functions, respectively.

TABLE I
VALUES OF R_c/R_r

$f(kc)$	$S°$	R_c/R_r Calculated by Cosine Integrals	R_c/R_r Calculated by Bessel Functions
200	22	2.757	2.784
300	33	2.46	2.527
400	44	2.10	2.193

It will be observed that for small spacings the values of R_c/R_r agree within 1 per cent; for greater spacing, they differ more. For $S = 360$ degrees the difference is about 10 per cent. It requires less time to compute R_c/R_r by (10) than using (6) and (7).

For small antenna spacings the coupled reactance for half-wave dipoles is positive; between values of S of 28 and 164 degrees it is negative oscillating with decreasing amplitude as S increases.

In the frequency band available for low-frequency

radio ranges and with the given antenna height and spacings, the total coupled reactance is small, always less than 10 ohms. This compares with a measured self-reactance for a single antenna of approximately 460 ohms at 400 kilocycles. For very small antenna spacings the reactance coupled by each antenna into the reference antenna approaches its self-reactance. In this case the inductance required to tune each antenna is approximately four times the inductance needed for tuning a

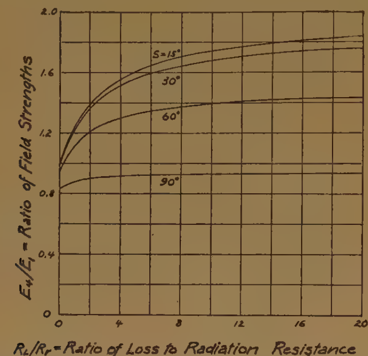


Fig. 3—Calculated gain in field strength obtainable by feeding a given power into four antennas instead of a single antenna versus R_L/R_r with S as parameter.

single antenna with the other antennas removed. Assuming a constant Q factor for the antenna-tuning coils, the power loss per antenna increases for very small antenna spacings over that for large spacing. The analysis presented here is restricted to such antenna spacings for which the coil resistance can be considered constant, i.e., to cases where the coupled reactance is small compared to the self-reactance of a single antenna.

It can be readily shown that, for a given frequency, input power, and antenna spacing, the field strength produced is nearly independent of the antenna height, provided the antenna height is smaller than one-quarter wavelength, and if coil and ground resistance and antenna (ohmic) resistance were negligible and impedance matching perfect.

Gain in Field Strength Due to Four Radiators over That with One Radiator for Constant Total Input Power

For a single antenna the field strength E_1 produced with an input power P_i is

$$E_1 = K\sqrt{P_i/(R_r + R_L)}. \tag{11}$$

For four antennas the root-mean-square field strength E_4 is

$$E_4 = K(E_{rms}/2)\sqrt{P_i/(R_r + R_L + R_c)} \tag{12}$$

where R_c and R_r are as defined for (10) and

R_L = coil and ground resistance for a single antenna

K = constant

E_{rms} = root-mean-square value of field strength for unit current in each antenna (by (3)).

The gain will then be the ratio of (11) and (12).

$$E_4/E_1 = E_{rms}/2\sqrt{(R_r + R_L)/(R_r + R_L + R_c)}$$
$$= E_{rms}/2\sqrt{(1 + \eta)/(1 + \eta + R_c/R_r)} \tag{13}$$

where $\eta = R_L/R_r$.

Under the above assumption that the coupled resistance R_c and radiation resistance R_r both be proportional to h^2, a plot of E_4/E_1 versus η holds for any antenna heights smaller than one-quarter wave. It must be remembered, however, that η varies with h in any practical case.

Equation (13) has been used to calculate the curves of Fig. 3 showing the gain in field strength (E_4/E_1) versus $\eta = R_L/R_r$ with one-half diagonal spacing S as parameter.

It is observed that for small spacings and a ratio R_L/R_r of approximtely five or over, a considerable gain in field strength can be obtained if the radio-frequency power is fed into four antennas instead of a single antenna. The power gain is proportional to the square of the field-strength gain and can be made of the order of 3:1.

In Fig. 4 the gain in field strength calculated by (13) is shown as a function of one-half diagonal antenna spacing S for a number of values of η. For values of S greater than about 80 degrees little or no gain can be obtained for any value of η. For a spacing $S = 98$ degrees all the curves intersect. This is because at this spacing the coupled resistance is zero and E_4 equals 1/2 E_{rms}. It is apparent that the greatest gain in field strength is obtained with highest coil and ground resistance and for small spacings. Spacings S in excess of approximately 45 degrees should not be chosen if maximum gain in field strength is desired. These conditions can readily be fulfilled in the present low-frequency radio ranges, as will be brought out below.

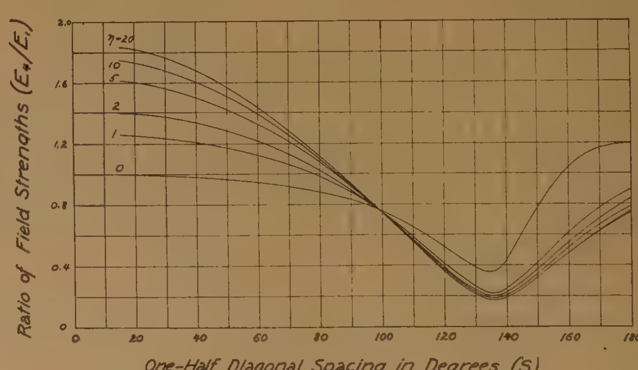


Fig. 4—Calculated gain in field strength obtainable by feeding a given power into four antennas instead of a single antenna versus S with $\eta = R_L/R_r$ as parameter.

In order to check the deviation from the desired circular field pattern, calculated field patterns (equation (1)) are shown in Table II for several spacings. Due to symmetry the field patterns need only be calculated between 0 and 45 degrees.

TABLE II

Relative Horizontal Field Strength

Degrees	S=44 degrees	S=85 degrees	S=135 degrees	S=270 degrees
0	3.438	2.174	0.586	2.0
10	3.438	2.15	0.570	1.229
20	3.434	2.10	0.190	−0.648
30	3.428	2.038	−0.139	−2.589
40	3.426	2.00	−0.351	−3.77
45	3.426	1.994	−0.376	−3.926
26.0			0	
16.9				0

Measurements

In October, 1934, measurements of field strengths obtained by energizing one and four antennas, respectively, were made by R. P. Battle at the Pittsburgh Pennsylvania, radio range. Measurements were made at 200 and 400 kilocycles at a location 0.82 mile north of the range station, in the direction 3 degrees off the bisector of the diagonals. The radio-frequency resistance (radiation, coil, and ground resistance) of individual towers was measured with the remaining three towers grounded. Thus the effect of coupled resistance was not present. The measured resistances for the four towers at 400 kilocycles were $R_{SE}=6.45$ ohms, $R_{NE}=6.6$ ohms, $R_{NW}=5.95$ ohms, and $R_{SW}=7.2$ ohms. With the southeast tower energized alone (the other towers grounded) radio-frequency current was 7.0 amperes, giving a field strength of 43,500 microvolts per meter.

Energizing all four towers simultaneously the currents were $I_{SE}=2.0$ amperes, $I_{NE}=2.0$ amperes, $I_{NW}=1.8$ amperes, and $I_{SW}=1.2$ amperes giving a field strength of 41,250 microvolts per meter.

For diagonal tower spacing $2S=600$ feet, we obtain at 400 kilocycles, $S=44$ degrees. The antennas are 125 feet high and have a radiation resistance of 1.04 ohms at 400 kilocycles as calculated by (8).

In order to find the total input power with four towers energized, the individual measured currents squared have to be multiplied by the sum of their measured and coupled resistances. The coupled resistance R_c is found by multiplying the ratio (R_c/R_r) (as calculated for quarter-wave radiators) by the radiation resistance of one 125-foot tower. To save the work of evaluating the real parts of (6) and (7), a curve of (R_c/R_r) versus S was drawn through a few calculated points (Fig. 5). From the curve we find for $S=44$ degrees, $R_c/R_r=2.12$.

The coupled resistance for each 125-foot tower is $R_c=2.12\times1.04=2.2$ ohms under the assumption of equal currents in all the towers.

The total input power into the four antennas is

	watts
$2.0^2\,(6.45+2.2)=$	34.6
$+1.8^2\,(5.95+2.2)=$	26.4
$+2.0^2\,(6.6\;+2.2)=$	35.2
$+1.2^2\,(7.2\;+2.2)=$	13.54
	109.74

The field strength for this condition was measured and found to be 41,250 microvolts per meter at 0.82 mile.

The power input with the southeast tower energized alone is $7.0^2\times6.45=316$ watts. The field strength for this condition was measured and found to be 43,500 microvolts per meter at 0.82 mile.

Adjusting the field strength with four towers energized to the same input power as with one tower energized gives $41{,}250\sqrt{316/109.7}=70{,}100$ microvolts per meter.

Measured gain $=70{,}100/43{,}500=1.61$.

The calculated gain is found by (13). For $S=44$ degrees,

$$E_{rms}=3.42$$

$$\eta=\frac{6.45+6.6+5.95+7.2-4\times1.04}{4\times1.04}=\frac{R_L}{R_r}$$

$$\eta=5.3$$

$$R_c/R_r=2.12$$

$$\frac{E_4}{E_1}=\frac{3.42}{2}\sqrt{\frac{1+5.3}{1+5.3+2.12}}.$$

Calculated gain $=1.48$.

The same measurements were repeated at 200 kilocycles.

$S=22$ degrees	$R_r=0.256$ ohm	$R_c/R_r=2.75$
$R_{SE}=5.63$ ohms	$I_{SE}=2.2$ amperes	
$R_{NE}=6.07$ ohms	$I_{NE}=2.1$ amperes	
$R_{NW}=5.97$ ohms	$I_{NW}=2.2$ amperes	
$R_{SW}=5.7$ ohms	$I_{SW}=2.0$ amperes	

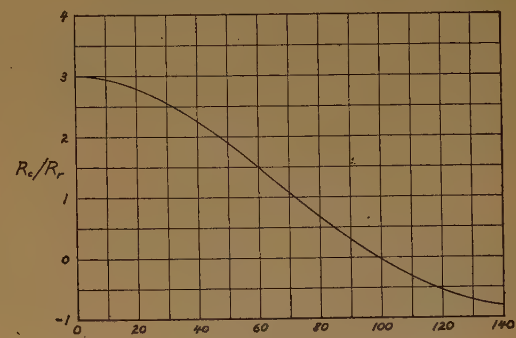


Fig. 5—Ratio of coupled resistance R_c to radiation resistance R versus S.

One tower energized:

$I_{SE}=7.05$ amperes yielding $E=21{,}150$ microvolts per meter

$P_i=280$ watts

Four towers energized:

$P_i=118.4$ watts yielding $E=28{,}500$ microvolts per meter

Adjusting the field strength with four towers energized to same input power as with one tower energized gives $\sqrt{280/118.4}\times28{,}500=43{,}800$ microvolts per meter.

Measured gain $=43{,}800/21{,}150=2.07$.

The calculated gain is obtained as follows:

$$E_{rms}=3.85$$

$$\eta=R_L/R_r=21.8$$

$$\frac{E_4}{E_1}=\frac{3.85}{2}\sqrt{\frac{22.8}{22.8+2.75}}=1.82.$$

Calculated gain $=1.82$.

It is observed that at 200 kilocycles the measured and calculated gains differ by 12 per cent.

At 400 kilocycles the agreement is better, the difference being about 8 per cent. In view of the several assumptions made in the analysis, the above agreements between measured and calculated gain are considered satisfactory.

Conclusions

With four vertical antennas shorter than one-quarter wavelength, located in the corners of a square, a considerable gain in field strength can be obtained over that produced by a similar single antenna into which the same total radio-frequency power is fed.

The gain obtainable is higher the larger the loss resistance (coil and ground resistance) and the smaller the spacing as long as the coupled reactance remains small relative to the self-reactance of a single antenna. No gain will be realized when the loss resistance is zero.

The spacing between diagonal towers ($2S$) must be smaller than approximately 170 degrees to realize any gain and to keep a substantially circular horizontal field pattern.

The analysis confirms that with ultra-high frequencies no gain is realized by feeding a given power in-phase into several antennas located symmetrically on a circle, over that of a single antenna. This is because the radiators are one-half or one-quarter wavelength long, requiring no loss resistance for tuning and grounding.

Acknowledgment

The writer is indebted to Mr. W. E. Jackson, Chief of the Radio Development Section, Civil Aeronautics Administration, for suggesting the problem and for valuable discussions pertaining to it.

Appendix

Derivation of Expression for R_c/R_r in Equation (10)

The idea used in this derivation is to set up equations for the radiated power of two identical short antennas in free space and of a single similar antenna by determining the root-mean-square field intensity in all directions of space in the two cases. Taking the ratio of the radiated powers gives the desired ratio of coupled resistance to radiation resistance for two antennas from which R_c/R_r for any number of similar antennas located symmetrically on a circle may be obtained. It can readily be shown that the above analysis holds also for short antennas over a perfectly conducting ground plane with the current increasing linearly from zero at the top to the maximum value at the base. This last procedure is straight forward and is not given here.

The radiation field intensity of two short antennas spaced a distance $2S$ and carrying equal currents in phase is

$$E = (120\pi Ih/\lambda r)\cos\phi\cos(S\sin\theta\cos\phi)$$
$$= 2E_{\max}\cos\phi\cos(S\sin\theta\cos\phi) \qquad (14)$$

where h = effective height
r = distance to reference point in space
ϕ = angle of elevation measured from equatorial plane
θ = azimuth angle measured from normal to line connecting the two antennas going through radiation center
$E_{\max}$ = maximum field intensity for a single antenna (in equatorial plane).

The root-mean-square field intensity of the two antennas taken along a circle of latitude ϕ is

$$E_{\text{rms hor}} = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} E^2 d\theta\right]^{1/2} \qquad (15)$$
$$= \left[\frac{4E_{\max}^2}{2\pi}\cos^2\phi\int_{-\pi}^{\pi}\cos^2(S\sin\theta\cos\phi)d\theta\right]^{1/2} \qquad (16)$$
$$E_{\text{rms hor}} = \sqrt{2}\,E_{\max}\cos\phi[1 + J_0(2S\cos\phi)]^{1/2}. \qquad (17)$$

To find the root-mean-square field intensity in all directions in space ($E_{\text{rms tot}}$) equation (17) is integrated in the meridian plane.

$$E_{\text{rms tot}} = \left[\frac{2}{\pi}\int_0^{\pi/2} E_{\text{rms}}{}^2{}_{\text{hor}} d\phi\right]^{1/2} \qquad (18)$$
$$E_{\text{rms tot}} = \left[\frac{4}{\pi}E_{\max}^2\int_0^{\pi/2}\cos^2\phi[1+J_0(2S\cos\phi)]d\phi\right]^{1/2}. \qquad (19)$$

Equation (19) is integrated in three parts S_1, S_2, and S_3 using the relation $\cos^2\phi = \frac{1}{2}(1+\cos 2\phi)$

$$S_1 = \frac{4}{\pi}E_{\max}^2\int_0^{\pi/2}\cos^2\phi d\phi = E_{\max}^2 \qquad (20)$$
$$S_2 = \frac{2}{\pi}E_{\max}^2\int_0^{\pi/2}J_0(2S\cos\phi)d\phi = E_{\max}^2 J_0^2(S) \qquad (21)$$
$$S_3 = \frac{2}{\pi}E_{\max}^2\int_0^{\pi/2}J_0(2S\cos\phi)\cos 2\phi d\phi$$
$$= -E_{\max}^2 J_1^2(S) \qquad (22)$$
$$E_{\text{rms tot}} = E_{\max}[1+J_0^2(S) - J_1^2(S)]^{1/2}. \qquad (23)$$

For a single antenna we may write with the same notation as above

$$E = E_{\max}\cos\phi$$
$$E_{\text{rms}} = \left[\frac{2}{\pi}E_{\max}^2\int_0^{\pi/2}\cos^2\phi d\phi\right]^{1/2} = \frac{1}{\sqrt{2}}E\max. \qquad (24)$$

The total power radiated by two antennas is

$$P_{r2} = 2I^2(R_r + R_c) \qquad (25)$$

with R_r = radiation resistance of a single antenna
R_c = coupled resistance
I = maximum current in each antenna.

For a single antenna with current I the radiated power is

$$P_{r1} = I^2R_r. \qquad (26)$$

Taking the ratio of (25) and (26) and introducing (23) and (24) we find

$$P_{r2}/P_{r1} = E_{\text{rms}}{}^2{}_{\text{tot}}/E_{\text{rms}}^2 = 2(1+R_c/R_r) \qquad (27)$$
$$R_c/R_r = (1/2)(E_{\text{rms}}{}^2{}_{\text{tot}}/E_{\text{rms}}^2) - 1 = J_0^2(S) - J_1^2(S). \qquad (28)$$

For four symmetrical antennas as used in radio ranges we may, therefore, write

$$R_c/R_r = 2[J_0^2(S/\sqrt{2}) - J_1^2(S/\sqrt{2})] + J_0^2(S) - J_1^2(S) \qquad (29)$$

which checks (10).

For the integrations in (21) and (22) the reader may consult, for example, the work by G. N. Watson, "A treatise on the theory of Bessel Functions," Cambridge, 1922, page 32.

Junction Analysis in Vacuum-Tube Circuits*

JOHN W. MILES†, NONMEMBER, I.R.E.

Summary—The method of junction analysis, well known to workers in circuit analysis, does not appear to be appreciated sufficiently by students and others whose interests do not lie primarily in electric-circuit theory. It is shown that considerable simplification and saving of computation is often effected when this method is used, as contrasted with the better known mesh analysis, especially in multistage amplifiers employing feedback, etc. Moreover, simpler representation of a vacuum tube is achieved.

I. Introduction

THE FACT that the analysis of a circuit may be made in terms of its junctions as well as its meshes has been expounded by a number of men in different fields;[1–4] nevertheless this approach does not appear to be sufficiently appreciated. It is the purpose of this paper to present the fundamentals and illustrate the simplification effected by junction analysis in some typical vacuum tube circuits. (This method of analysis also proves advantageous in circuits arising from electromechanical analogies.)

II. Junction Theory

It is assumed that the reader is familiar with the ordinary mesh analysis of circuits.[1,2,4] For a circuit of n meshes the current in mesh K is given by

$$i_k = \sum_{j=1}^{n} \frac{e_j C_{jk}}{\Delta} \qquad (k = 1, 2, \cdots, n) \tag{1}$$

where e_j is the net voltage generated around mesh j, Δ is the determinant formed by the self- and mutual impedances of the n meshes, and C_{jk} equal to $(-1)^{j+k}$ times the determinant obtained by eliminating the jth row and the kth column of Δ. (Keller[2] and Kron[3] express these results in the simpler notation of matrices.)

Assume now that our voltage generators are replaced by current generators and it is desired to find the potential at any point (junction) in the circuit. At any junction p we may write Kirchhoff's laws that the vector sum of the currents flowing into a junction is zero as

$$\sum_{q=1}^{n} (e_p - e_q) Y_{pq} = L_p \tag{2}$$

where e_p and e_q are the voltages of the junctions p and q with reference to some arbitrary junction (generally this will be the ground bus), Y_{pq} is the admittance between junctions p and q, and i_p is the net current injected at junction p by current generators at that junction. The term $q=p$ in (2) is, of course, zero. Equation (2) may be written for each of the n junctions of a circuit giving a set of n simultaneous linear equations in the n unknown voltages. (The ground bus or reference junction is not one of the n junctions; its *known* potential may be denoted by e_0, which is zero for a ground bus.) In the pth equation the coefficient of e_p will be the self-admittance Y_{pp},i .e., sum of all the admittances tied to the pth junction, while the coefficient of the e_q's will be the *negative* of the mutual admittance between junctions p and q (the dual of the mutual impedance between meshes). Very often the mutual admittance between two junctions is zero.

The solution of the simultaneous equations represented by (2) gives

$$e_p = \sum_{q=1}^{n} \frac{i_q C_{qp}}{\Delta} \qquad (p = 1, 2, \cdots, n) \tag{3}$$

where Δ is now the determinant of the self- and mutual-admittances of the n junctions, and C_{qp} is $(-1)^{q+p}$ times the determinant obtained by eliminating the qth row and the pth column of Δ. Note that (3) is completely analogous to (1) where voltages are interchanged with currents, impedances replace admittances, and junctions replace meshes. A familiar case of a junction replacing a mesh is the well-known star-delta transformation, and the general case is the covariant tensor transformation.[2,3]

A valuable variation in junction analysis results in the particular case of $i_p=0$; (2) may then be written as[5]

$$e_{p0} = \frac{\sum_{q=1}^{n} e_{q0} Y_{qp}}{\sum_{q=1}^{n} Y_{qp}}. \tag{4}$$

This theorem has been presented by Millman[6] and applied by him to three-phase circuits and to vacuum-tube circuits. It is particularly useful in unbalanced three-phase circuits since the voltage between sending and receiving neutrals is directly obtained; however, as will be made clear in the examples below, it is not so powerful as junction analysis in vacuum tube circuits. In the application of (4) it is clearly no longer necessary to convert voltage generators to current generators.

III. Application to Vacuum-Tube Analysis

The chief advantage of the junction method of circuit analysis is the availability of a solution involving a

* Decimal classification: R142. Original manuscript received by the Institute, August 2, 1943; revised manuscript received, May 23, 1944.

† Electrical Engineering Department, California Institute of Technology, Pasadena 4, California.

1 E. A. Guillemin, "Communication Networks," vol. 2, John Wiley and Sons, Inc., New York, N. Y., 1935.

2 E. G. Keller, "Mathematics of Modern Engineering," vol. 2, John Wiley and Sons, Inc., New York, N. Y., 1942.

3 Gabriel Kron, "Tensor Analysis of Networks," John Wiley and Sons, Inc., New York, N. Y., 1939.

4 Electrical Engineering Staff, Massachusetts Institute of Technology, "Electric Circuits," John Wiley and Sons, Inc., New York, N. Y., 1940.

5 The subscript 0 implies a common reference point for the potential e_{p0}.

6 Jacob Millman, "A useful network theorem," Proc. I.R.E., vol. 28, 9, pp. 413–418; September, 1940.

lower order determinant than mesh analysis might lead to. Accordingly a circuit should always be investigated to see whether it may be solved more simply in terms of junction voltages or mesh currents. Once these voltages or currents are found by determinant solution any other current or voltage is found by direct application of Ohm's law. A less important factor is whether the performance of the circuit is specified in terms of voltages. A circuit which has many impedance elements in parallel is almost always better solved by the junction method, for even though the loops formed by parallel impedances are not necessarily treated as separate meshes, it is far simpler to add parallel admittances than parallel impedances. A final, but all-important consideration is whether the circuit is more easily represented with current or voltage generators. The labor of converting an ideal voltage generator having zero series impedance to an ideal current generator having infinite shunt impedance and vice versa by Thevenin's theorem may be greater than the saving gained by the use of one method of analysis over the other.

In the case of the usual vacuum-tube circuit it may be said:

(1) There are fewer junctions than meshes.
(2) The performance of the circuit is generally specified in terms of voltage ratios.
(3) In the equivalent circuit many elements occur in parallel; and in particular there are usually more capacitances than inductances, facilitating the use of admittance.
(4) A vacuum tube may be represented equally well by a current or a voltage generator.[7]

Obviously the first three points favor junction analysis, while the fourth makes it simply applicable.

IV. Illustrations

(a) Consider the ordinary resistance coupled amplifier of Fig. 1. This amplifier and its equivalent circuit are given on page 175 of footnote reference (7) and is solved in footnote reference (5) by the application of (4). If e_s is the input voltage, and e_0 is the output voltage, the gain is given by the ratio of e_0 to e_s. We compute this gain first by the conventional mesh approach.

$$e_0 = i_2\left(\frac{z_5z_6}{z_5+z_6}\right) = \frac{\begin{vmatrix} (z_1 + (z_2z_3/z_2 + z_3)) & -\mu e_s \\ -(z_2z_3/z_2 + z_3)) & 0 \end{vmatrix} (z_5z_6/(z_5+z_6)}{\begin{vmatrix} (z_1 + (z_2z_3/z_2 + z_3)) & -(z_2z_3(z_2+z_3)) \\ -(z_2z_3/z_2+z_3) & (z_2z_3/z_2 + z_3 + z_4 + (z_5z_6/z_5+z_6)) \end{vmatrix}} \tag{5}$$

$$\left|\frac{e_0}{e_s}\right| = \mu\frac{z_{56}z_{23}}{(z_1+z_{23})(z_4+z_{56})+z_1z_{23}}. \tag{6}$$

Here Z_{56} is the impedance of Z_5 and Z_6 in parallel, etc. Millman[7] does away with the troublesome paralleled impedances in (6) by applying (4). If we let $Y_1 = 1/z_1$, $Y_2 = 1/z_2$, etc., then

$$e_0 = e_s'\frac{Y_4}{(Y_4+Y_5+Y_6)} \tag{7}$$

$$e_s' = \frac{e_0Y_4+\mu e_sY_1}{(Y_1+Y_2+Y_3+Y_4)} \tag{8}$$

$$\left|\frac{e_0}{e_s}\right| = \mu\frac{Y_1Y_4}{(Y_1+Y_2+Y_3+Y_4)(Y_4+Y_5+Y_6)-Y_4^2}. \tag{9}$$

The application of the junction analysis gives

$$e_0 = \frac{\begin{vmatrix} (Y_1+Y_2+Y_3+Y_4) & -g_me_s \\ -Y_4 & 0 \end{vmatrix}}{\begin{vmatrix} (Y_1+Y_2+Y_3+Y_4) & -Y_4 \\ -Y_4 & (Y_4+Y_5+Y_6) \end{vmatrix}} \tag{10}$$

$$\left|\frac{e_0}{e_s}\right| = \frac{g_mY_4}{(Y_1+Y_2+Y_3+Y_4)(Y_4+Y_5+Y_6)-Y_4^2}. \tag{11}$$

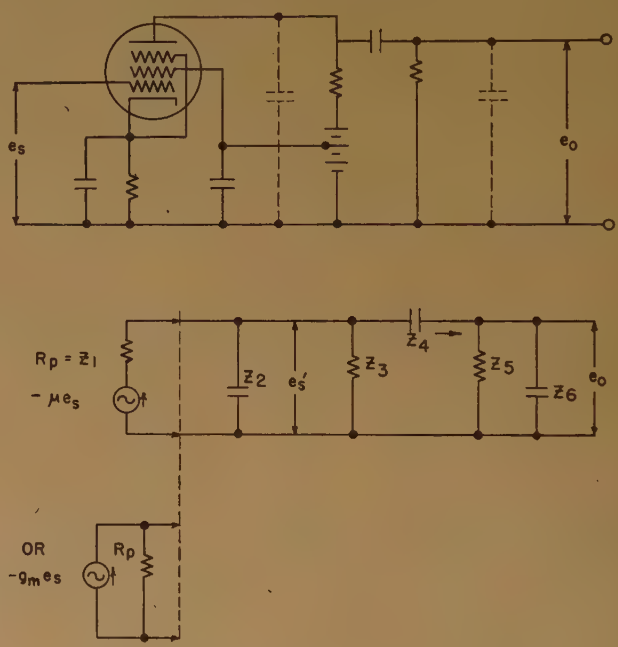


Fig. 1—An ordinary resistance-coupled amplifier.

Since $\mu Y_1 = g_m$, (9) and (11) are identical; however the solution leading to (11) is somewhat more direct.

(b) As a second illustration consider the two-stage resistance-capacitance-coupled amplifier incorporating feedback through R, as shown in Fig. 2. This particular circuit is solved by Millman through the use of (4). It requires 4 meshes or 3 junctions. For simplicity of illustration interelectrode capacitances are neglected; however, they do not add junctions to the circuit and could be easily included. If the coupling condenser is

[7] F. E. Terman, "Radio Engineering," McGraw-Hill Book Company, New York, N. Y., 1938.

large enough it too may be neglected, thus eliminating an extra junction or an extra mesh. In the usual approach to such a problem involving two stages it is not possible to write the solution of the output voltage directly since the grid-driving voltage of the second stage must be found. It will now be shown that the finding of this intermediate voltage is not necessary when the junction method of analysis is used.

Consider the Kirchhoff equations for the circuit

$$Y_{11}e_1 - Y_{12}e_2 - Y_{13}e_3 = -g_{m_1}e_s \qquad (12a)$$
$$-Y_{21}e_1 + Y_{22}e_2 - Y_{23}e_3 = 0 \qquad (12b)$$
$$-Y_{31}e_1 - Y_{32}e_2 + Y_{33}e_3 = -g_{m_2}e_2. \qquad (12c)$$

Generally all admittances are passive and bilinear so that $Y_{12}=Y_{21}$, etc. However if we rewrite (12c) as

$$-Y_{31}e_1 - (Y_{32} - g_{m_2})e_2 + Y_{33}e_3 = 0$$

or

$$-Y_{31}e_1 - Y'_{32}e_2 + Y_{33}e_3 = 0 \qquad (12c')$$

we see that the current generator of the second stage may be eliminated if only we define the mutual impedance from junction 3 to junction 2 (Y'_{32}) as the bilinear mutual impedance (Y_{23}) minus the transconductance of the amplifier tube whose control grid is at junction 2 and whose plate is at junction 3. The physical interpretation of this operation implies that a voltage at the grid of the tube produces a current in the plate circuit, but that a voltage at the plate produces no current in the grid circuit, this being a familiar and important property of the ordinary vacuum tube. Fig. 3 illustrates a possible representation of the tube where $-g_m$ does not contribute to the self-admittance of the plate junction. In general, however, there will be less confusion if this representation is introduced in the circuit equations rather than in the circuit schematic. This principle can be extended to cover the transconductances between other elements of a tube in cases where these elements have alternating-current potentials above ground. If there is any advantage the first tube may be similarly treated.

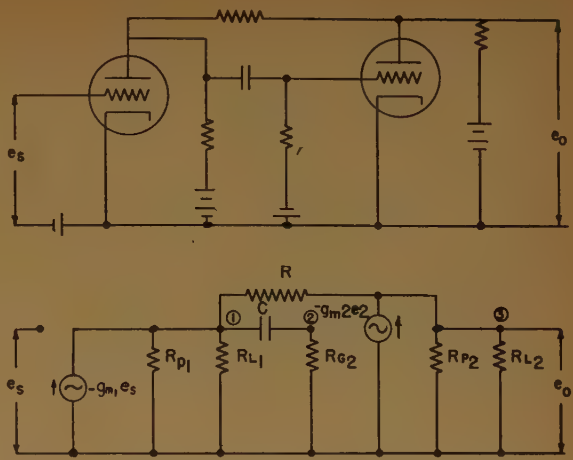


Fig. 2—A two-stage resistance-capacitance-coupled amplifier incorporating feedback through R.

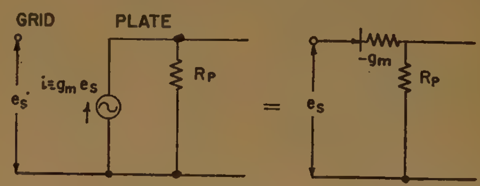


Fig. 3—The representation of a vacuum tube as a passive element.

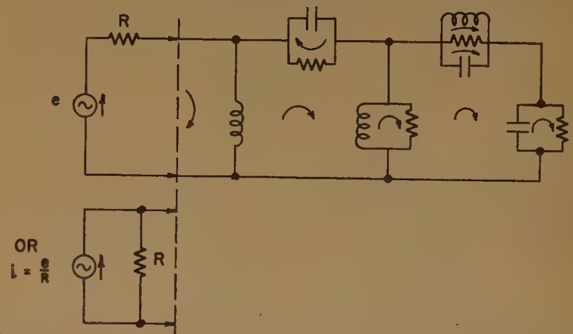


Fig. 4—A circuit which requires eight meshes for solution and only three nodes.

Returning to the solution of Fig. 2 we find the gain directly:

$$e_0 = \frac{\begin{vmatrix} -j\omega C & (j\omega C + 1/R_{G_2}) \\ -1/R & -g_{m_2} \end{vmatrix} (-g_{m_1}e_s)}{\begin{vmatrix} (1/R_{P_1} + 1/R_{L_1} + 1/R + j\omega C) & -j\omega C & -1/R \\ -j\omega C & (j\omega C + 1/R_{G_2}) & 0 \\ -1/R & -g_{m_2} & (1/R + 1/R_{P_2} + 1/R_{G_2}) \end{vmatrix}} \qquad (13)$$

$$\left|\frac{e_0}{e_s}\right| = g_{m_1}\left|\frac{(j\omega C g_{m_2} + j\omega C/R + 1/RR_{G_2})}{\Delta}\right|. \qquad (14)$$

If we had considered $j\omega C$ to have a large admittance, junction 2 would have been omitted, and we would have had a second-order determinant solution rather than a third. Note that it is not practicable in this particular case to let $j\omega C$ approach infinity in (13), although of course this may be done.

(c) The examples above have been simple in order to illustrate the method of solution. In Fig. 4 is shown a circuit (taken from page 429 of footnote reference 4) which in principle would require eight mesh currents, whereas it may be solved with three node voltages.[8] This circuit, however, brings us to an important limitation of junction analysis, for if there is coupling between any of the coils shown there is no direct way of handling the mutual impedance.

[8] Actually this circuit would be handled with three meshes, but the paralleling of impedance elements motivated by such an analysis would still leave the treatment considerably more complex than a function analysis.

Consider the transformer of Fig. 5(a). There are two meshes and the mutual inductance between the coils is treated as any other mutual impedance. If it is desired to use junction analysis it is necessary to replace it by its equivalent T (Fig. (5b)) or π (Fig. (5c)) network. The T equivalent is simply computed but possesses three nodes, while the π equivalent has only two nodes but more complex constants. The choice of π or T would obviously be a matter of the circuit and the constants of the transformer. Generally, however, circuits containing transformers are better handled with mesh currents.

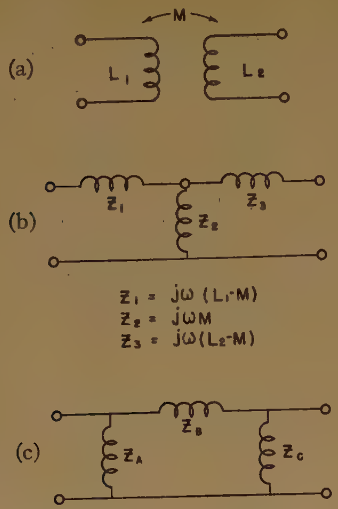


Fig. 5—A transformer and its T and π representations.

V.—Conclusion

From the above illustrations it is seen that the representation of a vacuum tube by a current generator in parallel with the plate impedances and the solution of the equivalent circuit in terms of its junction voltages generally leads to a simpler solution than the conventional mesh-current approach. The only important cases where this may not be true are in transformer-coupled amplifiers. A unique feature of the junction analysis allows vacuum tubes to be represented as passive networks composed of a bilinear self impedance and a *unilinear* mutual impedance.

The Calculation of the Mutual Inductance of Circular Filaments in Any Desired Positions*

FREDERICK W. GROVER†, SENIOR MEMBER, I.R.E.

***Summary*—The calculation of the mutual inductance of circular coils whose cross-sectional dimensions are small, compared with their distance apart, depends upon basic formulas for the calculation of the mutual inductance of circular filaments.**

The case of *coaxial* circular filaments has been quite thoroughly treated and any desired accuracy is attainable. However, the available formulas for more general cases such as circles with parallel or inclined axes are slowly convergent for considerable ranges of the parameters and for such cases admit of only rough accuracy.

The present paper is concerned with providing formulas and tables capable of giving a moderate accuracy, with a moderate amount of labor, even in complicated cases. The tables apply to calculations for equal circles with parallel axes, and to circles of nearly equal radii having inclined axes which intersect at the center of one of the circles. This latter case enters in the design of a variometer composed of two coils of small cross sections; the former is applicable to the calculation of a variometer with coils having eccentric axes.

A general formula is derived, which allows of a fairly simple method of computation for any desired case, by the employment of a chart of the flux distribution about a circular filament carrying current. An accuracy of a few parts in a thousand is readily attainable.

THE calculation of the mutual inductance of circular coils whose cross-sectional dimensions are small, compared with their distance apart, depends upon basic formulas for the calculation of the mutual inductance of circular filaments.

The case of *coaxial* circular filaments[1–5] has been quite thoroughly treated by a number of authors since the time of Maxwell and an accuracy exceeding anything required in practice is possible. Tables by Curtis and Sparks[6] and by Grover[7] make routine calculations quite simple and rapid. Calculations for more general cases have, however, received relatively little attention and only in special cases are satisfactory formulas available.

A formula for two circles whose axes intersect was given by Maxwell.[1] Formulas for circles with parallel axes have been given by Butterworth[8,9] and Snow.[10] Snow has also treated[11] the case of circles with inclined axes with the center of one circle on the axis of the other. Only in the case of Butterworth's formulas have any numerical data appeared. Unfortunately, existing formulas are slowly convergent, are useful for rather limited

* Decimal classification: R232. Original manuscript received by the Institute, April 17, 1944.

† Electrical Engineering Department, Union College, Schenectady, New York.

1 Maxwell, "Electricity and Magnetism," vol. 2, section 701.
2 E. B. Rosa and F. W. Grover, *Bull. Bur. Stand.*, vol. 8, pp. 1–237; January, 1912. Scientific Paper 169.
3 Butterworth, *Phil. Mag.*, vol. 31, p. 276; 1916.
4 F. W. Grover, *Bur. of Stand.* Scientific Paper 320, 1918.
5 F. W. Grover, *Bur. Stand. Jour. Res.*, vol. 1, pp. 487; 1928.
6 Curtis and Sparks, *Bull. Bur. Stand.*, vol. 19. Scientific Paper 492; 1924.
7 F. W. Grover, *Bull. Bur. Stand.*, vol. 20. Scientific Paper 498; 1924.
1 See section 697 of footnote reference 1.
8 Butterworth, *Phil. Mag.*, vol. 31, p. 443; 1916.
9 Scientific Paper 320, *Bur. of Stand.* 1918.
10 Chester Snow, *Bur. Stand. Jour. Res.*, vol. 3, p. 255; 1929.
11 Chester Snow, *Bur. Stand. Jour. Res.*, vol. 1, p. 531; 1928. Especially formula (23a).

ranges of the parameters, and fail completely in other cases. Furthermore, numerical computations are laborious because of the nature of the calculations and the number of terms which have to be calculated.

The present paper is concerned especially with the problem of providing methods and material suitable for routine calculations of an accuracy sufficient for practical purposes. Tables have been calculated for the more important special cases and a general method is developed capable of giving results accurate to a few parts in one thousand.

Circles with Parallel Axes

Let the axes of the circles be a and A (Fig. 1) and the distance between centers r. Butterworth's formula[9,10] for distant circles $r>(a+A)$ converges slowly unless r is appreciably greater than $1.5(a+A)$. Butterworth's

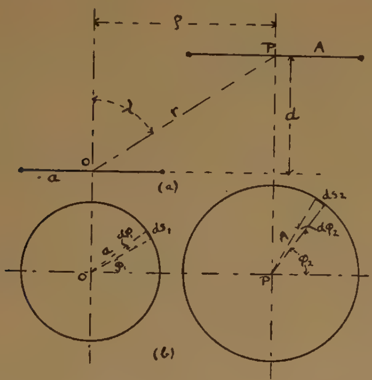


Fig. 1—Circular filaments with parallel axes.

formula for equal circles near together is useful only when r is less than the radius. Snow[10] shows that Butterworth's formula for unequal circles near together is incomplete. He provides the correct formula for the general case $(A-a)<r<(A+a)$, but in such a form that considerable labor is necessary to put it into a form suitable for computation. All these formulas are series expressions, each term involving shape parameters and zonal harmonics of the angle λ shown in Fig. 1; they are unsuited to routine numerical work.

The general formula for this case may be obtained by writing the Neumann integral $\int\int \cos \epsilon\, ds_1 ds_2/R$. Referring to Fig. 1 and taking as origin the center of one circle the co-ordinates of elements ds_1 and ds_2 of the circles are given by

$$\begin{aligned} x_1 &= a\cos\phi_1 & x_2 &= \rho + A\cos\phi_2 \\ y_1 &= a\sin\phi_1 & y_2 &= A\sin\phi_2 \\ z_1 &= 0 & z_2 &= d \\ ds_1 &= a\,d\phi_1 & ds_2 &= A\,d\phi_2. \end{aligned}$$

The angle ϵ between them is given by

$$\cos\epsilon = \frac{dx_1}{ds_1}\cdot\frac{dx_2}{ds_2} + \frac{dy_1}{ds_1}\cdot\frac{dy_2}{ds_2} + \frac{dz_1}{ds_1}\cdot\frac{dz_2}{ds_2} = \cos(\phi_1-\phi_2)$$

and the radius vector R by

$$R^2 = (\rho^2 + d^2 + a^2 + A^2) + 2\rho(A\cos\phi_2 - a\cos\phi_1) - 2aA\cos(\phi_1-\phi_2).$$

There results for the mutual inductance

$$M = aA\int_0^{2\pi} d\phi_1 \int_0^{2\pi} \frac{\cos(\phi_1-\phi_2)}{R}\, d\phi_2.$$

Performing the integration with respect to ϕ_2 there is found[12]

$$M = 4\pi\sqrt{Aa}\int_0^{\pi} \frac{d\phi_1}{\pi}\cdot\frac{(1-\rho/a\cos\phi_1)N}{V^{3/2}}, \tag{1}$$

in which

$$\begin{aligned} N &= (2/k - k)K - (2/k)E \\ V &= \sqrt{1 - 2(\rho/a)\cos\phi_1 + \rho^2/a^2} \end{aligned} \tag{2}$$

K and E are the complete elliptic integrals whose modulus k is given by the relation

$$1 - k^2 = k'^2 = \frac{(1-\alpha V)^2 + \delta^2}{(1+\alpha V)^2 + \delta^2} \tag{3}$$
$$\alpha = a/A, \qquad \delta = d/A.$$

The writer has not succeeded in performing the integration indicated in (1): attempts to develop the integrand in series form are blocked by the fact that both large and small values of k'^2 occur in the range of integration. However, it is entirely feasible to evaluate M by mechanical integration. The quantity $4\pi\sqrt{Aa}\cdot N$ is Maxwell's formula[1] for the mutual inductance of two coaxial circles and tables[7] give this in the form $f\sqrt{Aa}$ for values of f as a function of k'^2. Thus equation (1) may be written in the form

$$M = \sqrt{Aa}\int_0^{\pi} \frac{f}{\pi}\,\frac{(1-\rho/a\cos\phi)}{V^{3/2}}\, d\phi \text{ microhenries.} \tag{4}$$

The modulus k'^2 is calculated by (3) for the selected values of ϕ and the corresponding values of f taken from the tables[7] of the Bureau of Standards Scientific Paper 498. In performing the mechanical quadrature, the following procedure has been found effective. To apply Simpson's rule the interval of integration has to be divided into an even number of equal intervals. Weddle's rule assumes that the number of intervals is a multiple of six. If, therefore, the points are calculated for every 15 degrees the integration may be performed by both formulas. Simpson's rule ignores the fourth order in a table of differences calculated from the ordinates, while in Weddle's formula differences higher than the sixth are neglected. The closeness of agreement of the results found by the two formulas is a measure of the importance of higher-order differences. If now the interval of the ordinates is halved and a calculation is made for 24 points, then the difference between the two Simpson rule calculations, divided by 15, gives the correction to be applied to the 24-point value, provided differences higher than fourth order are unimportant. Furthermore, a close agreement between this corrected value and the value calculated by Weddle's rule from the 24 points confirms the accuracy of the result. The greatest difficulty lies in those cases where the curve of the ordinates passes steeply through a peak value. Values obtained from (4) using this procedure closely check the zonal

[12] Formulas (1) and (8), below were published by the author in *Communications*, October, 1938.

TABLE I

VALUES OF F IN FORMULA (5) FOR EQUAL CIRCLES WITH PARALLEL AXES

$\mu=$	$r/2a=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$r/2a=1$	μ
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0
0.9	1	1.0267	1.0330	1.0329	1.0265	1.0146	0.9982	0.9790	0.9584	0.9376	0.9176	0.9
0.8	1	1.0552	1.0692	1.0699	1.0568	1.0313	0.9953	0.9527	0.9070	0.8613	0.8180	0.8
0.7	1	1.0857	1.1087	1.1112	1.0919	1.0509	0.9917	0.9200	0.8428	0.7665	0.6959	0.7
0.6	1	1.1155	1.1517	1.1580	1.1328	1.0750	0.9876	0.8787	0.7619	0.6472	0.5441	0.6
0.5	1	1.1536	1.1997	1.2111	1.1812	1.1052	0.9842	0.8291	0.6585	0.4938	0.3515	0.5
0.4	1	1.1917	1.2524	1.2717	1.2390	1.1440	0.9836	0.7668	0.5246	0.2914	0.1014	0.4
0.3	1	1.2330	1.3109	1.3411	1.3085	1.1952	0.9897	0.6964	0.3849	0.0137	−0.2378	0.3
0.2	1	1.2780	1.3760	1.4212	1.3929	1.2641	1.0102	0.5850	0.1178	−0.3874	−0.7240	0.2
0.1	1	1.3274	1.4489	1.5139	1.4959	1.3577	1.0557	0.5505	−0.1681	−1.0231	−1.5087	0.1
0	1	1.3820	1.5311	1.6214	1.6220	1.4851	1.1450	0.5253	−0.4672	−1.953	−4.053	0
μ	$2a/r=1$	0.9	0.8	0.6	0.6	0.5	0.4	0.3	0.2	0.1	$2a/e=0$	μ
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0
0.9	0.9176	0.8968	0.8735	0.8481	0.8231	0.7946	0.7693	0.7471	0.7298	0.7188	0.7150	0.9
0.8	0.8180	0.7741	0.7266	0.6771	0.6292	0.5812	0.5398	0.5058	0.4806	0.4652	0.4600	0.8
0.7	0.6959	0.6267	0.5551	0.4848	0.4196	0.3626	0.3157	0.2797	0.2546	0.2399	0.2350	0.7
0.6	0.5441	0.4477	0.3543	0.2700	0.1992	0.1434	0.1019	0.0729	0.0540	0.0434	0.0400	0.6
0.5	0.3515	0.2275	0.1190	0.0336	−0.0287	−0.0697	−0.0956	−0.1109	−0.1195	−0.1237	−0.1250	0.5
0.4	0.1014	−0.0470	−0.1551	−0.2212	−0.2551	−0.2681	−0.2704	−0.2678	−0.2640	−0.2610	−0.2600	0.4
0.3	−0.2378	−0.3939	−0.4670	−0.4809	−0.4704	−0.4410	−0.4156	−0.3941	−0.3780	−0.3683	−0.3650	0.3
0.2	−0.7240	−0.8337	−0.7992	−0.7204	−0.6357	−0.5764	−0.5250	−0.4867	−0.4604	−0.4451	−0.4400	0.2
0.1	−1.5087	−1.3509	−1.0900	−0.8992	−0.7586	−0.6632	−0.5930	−0.5434	−0.5102	−0.4912	−0.4850	0.1
0	−4.053	−1.677	−1.2154	−0.9636	−0.8030	−0.6931	−0.6160	−0.5624	−0.5269	−0.5066	−0.5000	0

harmonic series formulas in cases where those are also applicable.

Table I has been calculated to facilitate routine calculations of the mutual inductance of circles with *equal radii* and axes parallel. The chosen parameters are $r/2a$ = distance between centers/diameter (or $2a/r$, whichever is less than unity), and $\mu = d/r = \cos\lambda$, (see Fig. 1). Writing M_0 for the mutual inductance of two coaxial circles having a spacing d equal to the value of r for the given circles, the required mutual inductance is

$$M = M_0 F \tag{5}$$

the quantity F being obtained from the table for the given values of the parameters, and M_0 from Table 4 or 6 of Scientific Paper 498 with the argument distance/diameter $= r/2a$ (or diameter/distance $= 2a/r$).

The values of F in Table I for $r/2a = 0.1$ to 0.5, inclusive, and for $2a/r = 0$ to 0.5 inclusive, were calculated by Butterworth's[9,10] zonal harmonics formulas. The remaining values had to be calculated by the quadrature formula, a time-consuming piece of work.

Example 1:

As an example, consider the case of two circles of equal radii $a = 15$ centimeters with a distance between centers $r = 20$ centimeters and the distance between their planes $d = 16$ centimeters so that $\mu = \cos\lambda = 0.8$. The value of $r/2a$ is 2/3. From Table 4 of Scientific Paper 498 there is found for equal circles for which distance/diameter = 2/3, the value $f = 0.0031239$, so that

$$m_0 = fa = 0.0031239(15) = 0.046858 \text{ microhenry.}$$

From Table 1, for $r/2a = 2/3$ and $\mu = 0.8$ the ratio $F = 0.9928$ is interpolated, so that

$$\begin{aligned} M &= 0.9928(0.04686) = 0.04652 \text{ microhenry} \\ &= 46.52 \text{ abhenries.} \end{aligned}$$

If each circle formed the center filament of a coil of 100 turns of very small cross section, the mutual inductance of the coils would be 100 times 100 as great as this value or 465.2 microhenries.

Example 2:

For the case of two circles each of 48 inches diameter, distance between planes 15 inches, and distance between centers $r = 50$ inches, the parameters are $2a/r = 48/50 = 0.96$ and $\mu = 15/50 = 0.3$. Table I gives $F = -0.3103$. From Table 6, Scientific Paper 498, with diameter/distance = 0.96, there is found $f = 0.0012982$, so that

$$\begin{aligned} M &= 24(2.54)(0.0012982)(-0.3103) \\ &= -0.02456 \text{ microhenries} \\ &= -24.56 \text{ abhenries.} \end{aligned}$$

The negative sign signifies that the electromotive force induced in one circle by a change of current in the other is opposite in direction to the electromotive force resulting from the same change of current with the circles arranged in the coaxial position.

An inspection of Table I brings out some interesting facts. With a given pair of equal circles, if one circle is moved, keeping the distance between centers constant and the axes parallel, the mutual inductance varies through a large range of values and in a manner depending upon the parameter $r/2a$. For circles near together, $r/2a$ small, the mutual inductance increases continuously from the coaxial to the coplanar position, owing to the decrease in distance between the planes. For the case $r/2a \cong 0.6$, the effect of decreasing distance between planes is compensated by the opposite effect of increasing distance between axes. For greater values of $r/2a$, the mutual inductance decreases continuously with increasing λ, that is, decreasing μ, passes through zero and becomes negative. Table I is useful in placing two circles or coils with parallel axes so as to have zero mutual inductance. For values of $r/2a$ less than about 0.76, the mutual inductance does not become zero for any value of the angle λ. Table II shows values of λ for zero mutual inductance for different spacings.

Table I suffices for all cases of equal circles with parallel axes where values may be interpolated with the required accuracy. Otherwise, the solution may be made by direct calculation by (5) or by the general method to

TABLE II

ANGULAR POSITION FOR ZERO MUTUAL INDUCTANCE, PARALLEL EQUAL CIRCLES

$2a/r$	μ	λ_0	$2a/r$	μ	λ_0	$r/2a$	μ	λ_0
		Degrees			Degrees			Degrees
0	0.58	54.6	0.5	0.53	58.0	1.0	0.37	68.3
0.1	0.58	54.6	0.6	0.51	59.3	0.9	0.30	72.5
0.2	0.575	54.9	0.7	0.485	61.0	0.8	0.16	80.8
0.3	0.565	55.6	0.8	0.455	62.9	—	—	—
0.4	0.55	56.6	0.9	0.42	65.2	—	—	—
0.5	0.53	58.0	1.0	0.37	68.3	—	—	—

be described later. An exception is made of the case of equal coplanar circles. These will require special treatment, since interpolation for them in Table I is uncertain.

For equal intersecting coplanar circles write

$$M = Ca \text{ microhenries} \tag{6}$$

and for distant coplanar circles

$$M = -0.001(\pi^2/8)(2a/r)^3 D \text{ microhenries.} \tag{7}$$

Values of C and D are given in Table III.

TABLE III

CONSTANTS IN FORMULAS (6) AND (7). EQUAL COPLANAR CIRCLES

$r/2a$	C	Diff.	$\log_{10} C$	$2a/r$	C	D	Diff.
0.1	0.029766		$\bar{2}.47372$	1.0	−0.005749	4.6604	—
0.2	0.020681	−9085	$\bar{2}.31557$	0.9	−0.001886	2.0969	
0.3	0.015073	−5608	$\bar{2}.17821$	0.8	−0.001041	1.6482	−4487
0.4	0.010840	−4233	$\bar{2}.03505$	0.7	−0.0006160	1.4132	−2350
0.5	0.007334	−3506	$\bar{3}.86537$	0.6	−0.0003376	1.2669	−1463
0.6	0.004272	−3062	$\bar{3}.63068$	0.5	−0.0001802	1.1686	− 983
0.7	+0.001506	−2766	$\bar{3}.17776$	0.4	—	1.1006	− 680
0.8	−0.001045	−2551	$n\bar{3}.01915$	0.3	—	1.0538	− 468
0.9	−0.003457	−2412	$n\bar{3}.53869$	0.2	—	1.0232	− 306
1.0	−0.005749	−2292	$n\bar{3}.75964$	0.1	—	1.0057	− 175
				0		1	− 57

Tables for unequal circles with parallel axes would have to be based on three parameters and would be voluminous. It seems better to calculate individual cases as they arise from (5) or, where possible, from an appropriate series formula. For routine purposes where a moderate accuracy only is required the general method described below is to be recommended.

CIRCLES WITH INCLINED AXES

The most important case is that in which the intersection of the axes is at the center of one of the circles. The nomenclature is shown in Fig. 2. For circles where the ratio of the radii $\alpha = a/A$ is nearly unity and the distance ratio $\delta = d/A$ is small, the series expression of Snow,[11] which involves zonal harmonics $P_m(\mu)$ and the derivatives $P_m'(\delta/\sqrt{1+\delta^2})$, converges slowly. Such cases may be treated by writing the Neumann integral for inclined circles, following the method described for the previous case. The resultant expression is

$$M = \sqrt{Aa} \cdot \cos\theta \int_0^\pi \frac{N}{\pi} \frac{d\phi}{P^{3/2}} \tag{8}$$

in which again N is to be identified with the quantity $1000\,f$ in Table 1 of Scientific Paper 498 for coaxial circles, but the argument now is

$$k'^2 = \frac{1+\alpha^2+\delta^2+2\alpha\delta\cos\phi\sin\theta - 2\alpha P}{1+\alpha^2+\delta^2+2\alpha\delta\cos\phi\sin\theta + 2\alpha P} \tag{9}$$

with

$$P = \sqrt{1-\cos^2\phi\sin^2\theta}. \tag{10}$$

Formula (8) may be evaluated by mechanical quadrature as in the previous case. It gives results in agreement with Snow's formula in the region where the latter converges rapidly.

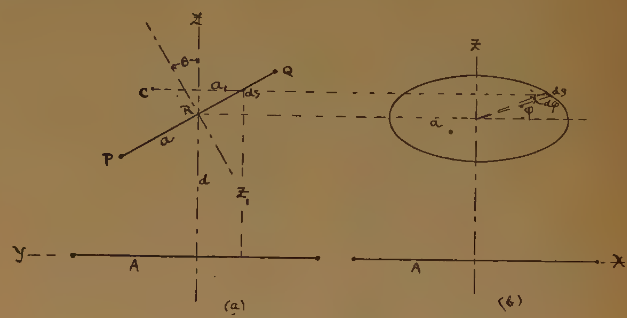


Fig. 2—Circular filaments with inclined axes.

Table IV has been computed to facilitate routine calculations. This covers the more difficult but important cases of nearly equal circles, values of the ratio of the radii α from 0.5 to 0.9 being included. The parameters used are $\mu = \cos\theta$ and $\delta = d/A$ (or $1/\delta$, whichever is less than unity). For $\alpha = 0.8$ and $\alpha = 0.9$ and the smaller values of δ the mechanical quadrature formula (8) had to be employed. In the rest of the cases Snow's formula was employed, but even here the labor was quite heavy, since terms of the order of those involving $P_{15}(\mu)$ were included where necessary.

In Table IV, as in Table I, the calculation of the mutual inductance is based on the corresponding case of coaxial circles. The quantity tabulated is the factor F_1 in the formula

$$M = M_0 F_1 \cos\theta. \tag{11}$$

This form of expression takes account of the fact that if the field were uniform, the mutual inductance would be proportional to the cosine of the angle of inclination of the axes. M_0, as before, is the mutual inductance of coaxial circles having the same radii and the same centers as the given circles. F_1 is a factor which takes into account the nonuniformity of the field. For smaller values of α not included in the table this factor may be calculated by the following formula derived from Snow's[11] expression

$$F_1 = \frac{1 - (1/4)\beta^2 P_3'(\gamma)P_3(\mu)/\mu + (1/8)(\beta^4)P_5'(\gamma)P_5(\mu)/\mu - (5/64)\beta^6 P_7'(\gamma)P_7(\mu)/\mu + \cdots}{1 - (1/4)\beta^2 P_3'(\gamma) + (1/8)\beta^4 P_5'(\gamma) - (5/64)\beta^6 P_7'(\gamma) + \cdots} \tag{12}$$

TABLE IV

VALUES OF F_1 IN FORMULA (11) FOR CIRCLES WITH INCLINED AXES

RATIO OF RADII = 0.5

μ	$1/\delta=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	μ
0	1	1.0092	1.0349	1.0719	1.1129	1.1496	1.1793	1.1965	1.2019	1.1988	1.1852	0
0.1	1	1.0091	1.0346	1.0712	1.1116	1.1489	1.1774	1.1944	1.1999	1.1969	1.1837	0.1
0.2	1	1.0088	1.0335	1.0688	1.1080	1.1439	1.1715	1.1882	1.1938	1.1912	1.1790	0.2
0.3	1	1.0084	1.0317	1.0650	1.1018	1.1357	1.1618	1.1778	1.1837	1.1816	1.1710	0.3
0.4	1	1.0077	1.0292	1.0598	1.0934	1.1243	1.1483	1.1634	1.1694	1.1681	1.1596	0.4
0.5	1	1.0069	1.0260	1.0530	1.0827	1.1099	1.1312	1.1450	1.1511	1.1507	1.1442	0.5
0.6	1	1.0059	1.0221	1.0449	1.0698	1.0927	1.1107	1.1227	1.1286	1.1292	1.1247	0.6
0.7	1	1.0047	1.0175	1.0355	1.0550	1.0728	1.0870	1.0968	1.1020	1.1034	1.1007	0.7
0.8	1	1.0033	1.0123	1.0248	1.0382	1.0506	1.0605	1.0674	1.0714	1.0734	1.0719	0.8
0.9	1	1.0017	1.0065	1.0130	1.0199	1.0262	1.0313	1.0350	1.0373	1.0390	1.0382	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

μ	$\delta=1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	$\delta=0$	μ
0	1.1852	1.1660	1.1376	1.0993	1.0522	0.9966	0.9407	0.8835	0.8361	0.8045	0.7934	0
0.1	1.1837	1.1648	1.1368	1.0989	1.0523	0.9972	0.9414	0.8841	0.8373	0.8058	0.7947	0.1
0.2	1.1790	1.1612	1.1343	1.0979	1.0526	0.9989	0.9434	0.8873	0.8410	0.8097	0.7987	0.2
0.3	1.1710	1.1548	1.1300	1.0958	1.0529	1.0016	0.9468	0.8928	0.8472	0.8164	0.8054	0.3
0.4	1.1596	1.1455	1.1234	1.0925	1.0529	1.0050	0.9518	0.9007	0.8563	0.8262	0.8154	0.4
0.5	1.1442	1.1328	1.1143	1.0874	1.0523	1.0088	0.9582	0.9111	0.8686	0.8394	0.8289	0.5
0.6	1.1247	1.1161	1.1016	1.0797	1.0502	1.0125	0.9663	0.9242	0.8844	0.8568	0.8468	0.6
0.7	1.1007	1.0950	1.0847	1.0685	1.0457	1.0153	0.9757	0.9401	0.9046	0.8794	0.8701	0.7
0.8	1.0719	1.0688	1.0627	1.0524	1.0372	1.0159	0.9858	0.9587	0.9299	0.9087	0.9008	0.8
0.9	1.0382	1.0372	1.0346	1.0300	1.0227	1.0121	0.9944	0.9795	0.9614	0.9474	0.9420	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

RATIO OF RADII = 0.6

μ	$1/\delta=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$1/\delta=1$	μ
0	1	1.0133	1.0507	1.1052	1.1665	1.2234	1.2670	1.2925	1.2998	1.2942	1.2722	0
0.1	1	1.0131	1.0502	1.1040	1.1646	1.2208	1.2640	1.2894	1.2968	1.2913	1.2702	0.1
0.2	1	1.0127	1.0486	1.1006	1.1590	1.2131	1.2549	1.2800	1.2880	1.2826	1.2641	0.2
0.3	1	1.0121	1.0459	1.0949	1.1496	1.2005	1.2400	1.2643	1.2731	1.2690	1.2535	0.3
0.4	1	1.0111	1.0422	1.0870	1.1368	1.1830	1.2194	1.2425	1.2521	1.2505	1.2378	0.4
0.5	1	1.0100	1.0375	1.0770	1.1206	1.1651	1.1934	1.2147	1.2247	1.2253	1.2163	0.5
0.6	1	1.0085	1.0319	1.0650	1.1014	1.1351	1.1624	1.1810	1.1910	1.1933	1.1882	0.6
0.7	1	1.0067	1.0252	1.0510	1.0794	1.1055	1.1268	1.1420	1.1510	1.1537	1.1526	0.7
0.8	1	1.0047	1.0177	1.0356	1.0549	1.0727	1.0874	1.0983	1.1053	1.1087	1.1090	0.8
0.9	1	1.0025	1.0092	1.0185	1.0283	1.0373	1.0449	1.0506	1.0546	1.0570	1.0579	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

μ	$\delta=1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	$\delta=0$	μ
0	1.2722	1.2425	1.1996	1.1431	1.0754	0.9970	0.9166	0.8404	0.7746	0.7306	0.7149	0
0.1	1.2702	1.2411	1.1988	1.1430	1.0758	0.9979	0.9179	0.8418	0.7761	0.7321	0.7164	0.1
0.2	1.2641	1.2366	1.1963	1.1425	1.0771	1.0006	0.9216	0.8461	0.7806	0.7366	0.7209	0.2
0.3	1.2535	1.2288	1.1918	1.1413	1.0790	1.0051	0.9278	0.8533	0.7883	0.7444	0.7287	0.3
0.4	1.2378	1.2169	1.1844	1.1388	1.0810	1.0111	0.9366	0.8639	0.7996	0.7560	0.7403	0.4
0.5	1.2163	1.2000	1.1732	1.1340	1.0826	1.0183	0.9480	0.8779	0.8150	0.7718	0.7563	0.5
0.6	1.1882	1.1768	1.1566	1.1253	1.0823	1.0258	0.9619	0.8960	0.8354	0.7932	0.7779	0.6
0.7	1.1526	1.1460	1.1327	1.1106	1.0779	1.0323	0.9777	0.9187	0.8621	0.8218	0.8071	0.7
0.8	1.1090	1.1064	1.0995	1.0866	1.0660	1.0344	0.9936	0.9460	0.8972	0.8608	0.8472	0.8
0.9	1.0579	1.0575	1.0554	1.0505	1.0417	1.0260	1.0048	0.9764	0.9429	0.9160	0.9055	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

RATIO OF RADII = 0.7

μ	$1/\delta=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$1/\delta=1$	μ
0	1	1.0181	1.0696	1.1459	1.2331	1.3150	1.3777	1.4136	1.4221	1.4080	1.3788	0
0.1	1	1.0179	1.0689	1.1442	1.2303	1.3112	1.3733	1.4091	1.4182	1.4049	1.3765	0.1
0.2	1	1.0174	1.0667	1.1393	1.2220	1.2999	1.3601	1.3958	1.4062	1.3951	1.3693	0.2
0.3	1	1.0165	1.0630	1.1311	1.2084	1.2813	1.3383	1.3735	1.3858	1.3783	1.3565	0.3
0.4	1	1.0152	1.0578	1.1199	1.1899	1.2558	1.3083	1.3422	1.3566	1.3534	1.3370	0.4
0.5	1	1.0135	1.0513	1.1058	1.1666	1.2240	1.2704	1.3021	1.3180	1.3193	1.3091	0.5
0.6	1	1.0115	1.0435	1.0889	1.1393	1.1866	1.2256	1.2536	1.2698	1.2749	1.2708	0.6
0.7	1	1.0092	1.0343	1.0697	1.1083	1.1446	1.1749	1.1976	1.2125	1.2197	1.2205	0.7
0.8	1	1.0064	1.0240	1.0482	1.0744	1.0988	1.1194	1.1354	1.1469	1.1540	1.1574	0.8
0.9	1	1.0034	1.0125	1.0249	1.0381	1.0502	1.0606	1.0689	1.0752	1.0795	1.0827	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

μ	$\delta=1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	$\delta=0$	μ
0	1.3788	1.3354	1.2745	1.1967	1.1055	1.0028	0.8979	0.7974	0.7104	0.6497	0.6276	0
0.1	1.3765	1.3339	1.2740	1.1970	1.1066	1.0043	0.8995	0.7990	0.7121	0.6513	0.6291	0.1
0.2	1.3693	1.3293	1.2721	1.1977	1.1097	1.0088	0.9044	0.8041	0.7172	0.6562	0.6337	0.2
0.3	1.3565	1.3210	1.2684	1.1985	1.1140	1.0159	0.9127	0.8129	0.7259	0.6645	0.6419	0.3
0.4	1.3370	1.3076	1.2618	1.1985	1.1197	1.0255	0.9247	0.8258	0.7387	0.6769	0.6542	0.4
0.5	1.3091	1.2872	1.2503	1.1961	1.1253	1.0375	0.9406	0.8434	0.7566	0.6943	0.6714	0.5
0.6	1.2708	1.2571	1.2308	1.1886	1.1293	1.0511	0.9607	0.8667	0.7807	0.7182	0.6950	0.6
0.7	1.2205	1.2144	1.1992	1.1712	1.1275	1.0641	0.9846	0.8971	0.8135	0.7512	0.7279	0.7
0.8	1.1574	1.1568	1.1512	1.1375	1.1125	1.0702	1.0097	0.9354	0.8585	0.7984	0.7753	0.8
0.9	1.0827	1.0842	1.0840	1.0808	1.0729	1.0557	1.0251	0.9790	0.9217	0.8709	0.8503	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

RATIO OF RADII = 0.8

μ	$1/\delta=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$1/\delta=1$	μ
0	1	1.0237	1.0919	1.1947	1.3139	1.4287	1.5156	1.5633	1.5718	1.5492	1.5063	0
0.1	1	1.0235	1.0909	1.1924	1.3100	1.4232	1.5093	1.5573	1.5668	1.5455	1.5039	0.1
0.1	1	1.0228	1.0879	1.1856	1.2983	1.4070	1.4908	1.5393	1.5517	1.5342	1.4964	0.2
0.3	1	1.0216	1.0830	1.1743	1.2791	1.3805	1.4600	1.5089	1.5254	1.5142	1.4827	0.3
0.4	1	1.0199	1.0761	1.1589	1.2531	1.3444	1.4177	1.4658	1.4868	1.4833	1.4608	0.4
0.5	1	1.0177	1.0674	1.1396	1.2209	1.2997	1.3645	1.4101	1.4346	1.4393	1.4272	0.5
0.6	1	1.0151	1.0569	1.1169	1.1833	1.2477	1.3018	1.3424	1.3682	1.3795	1.3777	0.6
0.7	1	1.0120	1.0449	1.0911	1.1414	1.1901	1.2317	1.2645	1.2881	1.3026	1.3085	0.7
0.8	1	1.0084	1.0313	1.0627	1.0961	1.1285	1.1562	1.1791	1.1969	1.2101	1.2187	0.8
0.9	1	1.0044	1.0162	1.0322	1.0485	1.0646	1.0782	1.0897	1.0991	1.1067	1.1128	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

TABLE IV—*Continued*

μ	$\delta=1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	$\delta=0$	μ
0	1.5063	1.4454	1.3637	1.2627	1.1475	1.0189	0.8911	0.768	0.655	0.566	0.530	0
0.1	1.5039	1.4443	1.3640	1.2636	1.1492	1.0210	0.8928	0.767	0.654	0.568	0.533	0.1
0.2	1.4964	1.4408	1.3636	1.2664	1.1540	1.0271	0.8985	0.773	0.658	0.573	0.536	0.2
0.3	1.4827	1.4337	1.3627	1.2706	1.1617	1.0368	0.9090	0.782	0.667	0.578	0.543	0.3
0.4	1.4608	1.4209	1.3593	1.2752	1.1721	1.0508	0.9245	0.796	0.680	0.592	0.556	0.4
0.5	1.4272	1.3989	1.3503	1.2782	1.1845	1.0693	0.9453	0.819	0.700	0.611	0.573	0.5
0.6	1.3777	1.3627	1.3304	1.2754	1.1963	1.0916	0.9729	0.847	0.729	0.633	0.597	0.6
0.7	1.3085	1.3059	1.2913	1.2583	1.2015	1.1153	1.0077	0.886	0.765	0.668	0.627	0.7
0.8	1.2187	1.2235	1.2231	1.2126	1.1855	1.1309	1.0475	0.928	0.818	0.719	0.679	0.8
0.9	1.1128	1.1178	1.1222	1.1242	1.1215	1.1069	1.0719	1.0075	0.905	0.807	0.764	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

Ratio of Radii = 0.9

μ	$1/\delta=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	$1/\delta=1$	μ
0	1	1.0305	1.1178	1.2526	1.4133	1.5681	1.6850	1.7454	1.7477	1.7158	1.6564	0
0.1	1	1.0301	1.1165	1.2495	1.4079	1.5605	1.6765	1.7379	1.7420	1.7120	1.6541	0.1
0.1	1	1.0292	1.1126	1.2403	1.3916	1.5379	1.6514	1.7148	1.7242	1.7003	1.6470	0.2
0.3	1	1.0276	1.1061	1.2251	1.3651	1.5010	1.6091	1.6751	1.6926	1.6787	1.6343	0.3
0.4	1	1.0255	1.0971	1.2045	1.3294	1.4510	1.5508	1.6178	1.6443	1.6437	1.6131	0.4
0.5	1	1.0227	1.0858	1.1789	1.2857	1.3897	1.4777	1.5425	1.5765	1.5899	1.5765	0.5
0.6	1	1.0193	1.0723	1.1490	1.2353	1.3192	1.3922	1.4501	1.4875	1.5118	1.5155	0.6
0.7	1	1.0153	1.0568	1.1154	1.1800	1.2423	1.2975	1.3438	1.3783	1.4066	1.4221	0.7
0.8	1	1.0107	1.0394	1.0789	1.1213	1.1618	1.1977	1.2291	1.2546	1.2781	1.2955	0.8
0.9	1	1.0056	1.0204	1.0402	1.0609	1.0802	1.0972	1.1125	1.1248	1.1374	1.1478	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0
μ	$\delta=1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	$\delta=0$	μ
0	1.6564	1.5737	1.4700	1.3451	1.2034	1.0588	0.9025	0.7511	0.6078	0.4741	0.4099	0
0.1	1.6541	1.5734	1.4705	1.3466	1.2057	1.0580	0.9050	0.7520	0.6095	0.4763	0.4114	0.1
0.2	1.6470	1.5722	1.4725	1.3517	1.2130	1.0644	0.9109	0.7596	0.6150	0.4807	0.4161	0.2
0.3	1.6343	1.5686	1.4762	1.3606	1.2260	1.0808	0.9273	0.7725	0.6246	0.4890	0.4229	0.3
0.4	1.6131	1.5598	1.4793	1.3723	1.2427	1.0973	0.9432	0.7887	0.6389	0.5012	0.4311	0.4
0.5	1.5765	1.5403	1.4771	1.3845	1.2665	1.1207	0.9693	0.8121	0.6593	0.5185	0.4472	0.5
0.6	1.5155	1.5008	1.4620	1.3921	1.2887	1.1564	1.0067	0.8473	0.6886	0.5431	0.4673	0.6
0.7	1.4221	1.4275	1.4178	1.3820	1.3091	1.2007	1.0567	0.8960	0.7308	0.5794	0.4969	0.7
0.8	1.2955	1.3100	1.3214	1.3225	1.2993	1.2331	1.1190	0.9648	0.7950	0.6383	0.5433	0.8
0.9	1.1478	1.1579	1.1698	1.1829	1.1971	1.1918	1.1694	1.0658	0.9064	0.7313	0.6278	0.9
1.0	1	1	1	1	1	1	1	1	1	1	1	1.0

in which

$$\beta^2 = \frac{\alpha^2}{1+\delta^2} = \frac{\alpha^2(1/\delta)^2}{1+(1/\delta)^2} \tag{13}$$
$$\gamma^2 = \frac{\delta^2}{1+\delta^2} = \frac{1}{1+(1/\delta)^2}$$

The general term of the denominator is

$$(-1)\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n(n+1)}\beta^{2n}P'_{2n+1}(\gamma) \tag{14}$$

and $P_{2n+1}'(\gamma)$ denotes the differential coefficient of the corresponding zonal harmonic with respect to its argument. The zonal harmonic factors may be taken from a table. The following series, however, may be useful

$$\begin{aligned}
P_3'(\gamma) &= 3/2(5\gamma^2-1),\\
P_5'(\gamma) &= 15/8(21\gamma^4-14\gamma^2+1)\\
P_7'(\gamma) &= 7/16(429\gamma^6-495\gamma^4+135\gamma^2-5)\\
P_3(\mu)/\mu &= 1/2(5\mu^2-3),\\
P_5(\mu)/\mu &= 1/8(63\mu^4-70\mu^2+15)\\
P_7(\mu)/\mu &= 1/16(429\mu^6-693\mu^4+315\mu^2-35).
\end{aligned} \tag{15}$$

In (11), F_1 is quite closely equal to unity for very distant circles, where the magnetic field is approximately uniform over the area of the circle. For nearer circles it may be either greater or less than unity. An examination of Fig. 2 shows that, for the region P of the smaller circle, the magnetic flux density is greater than at the center R, and at Q the flux density is less than at R. These differences nearly compensate for values of δ of the order of 0.5. For the concentric case $\delta=0$ both regions P and Q are in much weaker portions of the field than that at the center and F_1 is considerably smaller than unity.

A study of Table IV shows that the mutual inductance of a variometer consisting of two circular coils of small cross section, with the movable coil centered on the axis of the larger, would have a mutual inductance closely proportional to the cosine of the angular inclination, even with quite a range of values of the ratio α of the radii, provided the space ratio parameter δ is of the order of 0.5 or a little smaller. To obtain a more nearly linear relation, the coils should be placed still more nearly concentric. (It is, of course, impossible to obtain a linear range over the whole 90 degrees of rotation.) To obtain larger values of mutual inductance, the radii of the coils should approach equality. However, in such cases the effect of the cross sectional area would become more important.

General Method of Calculation

Tables to cover all possible cases of inclined circles and unequal circles with parallel axes and with intervals of the argument small enough to allow of accurate interpolation, would be very unwieldy and would entail a vast amount of calculation for their preparation. Furthermore, other more general cases would remain unprovided for. The following general method for making routine calculations in *any* desired case is offered as a practical solution of the problem. It leads to an accuracy sufficient for most practical work and with no excessive amount of labor.

A study of (4) and (8) shows that in each the mutual inductance is given as a line integral, each element of which may be evaluated in terms of the mutual inductance of a pair of coaxial circles. Consider, for greater definiteness, the case of the inclined circles in Fig. 2. Taking the origin at the center of the larger circle,

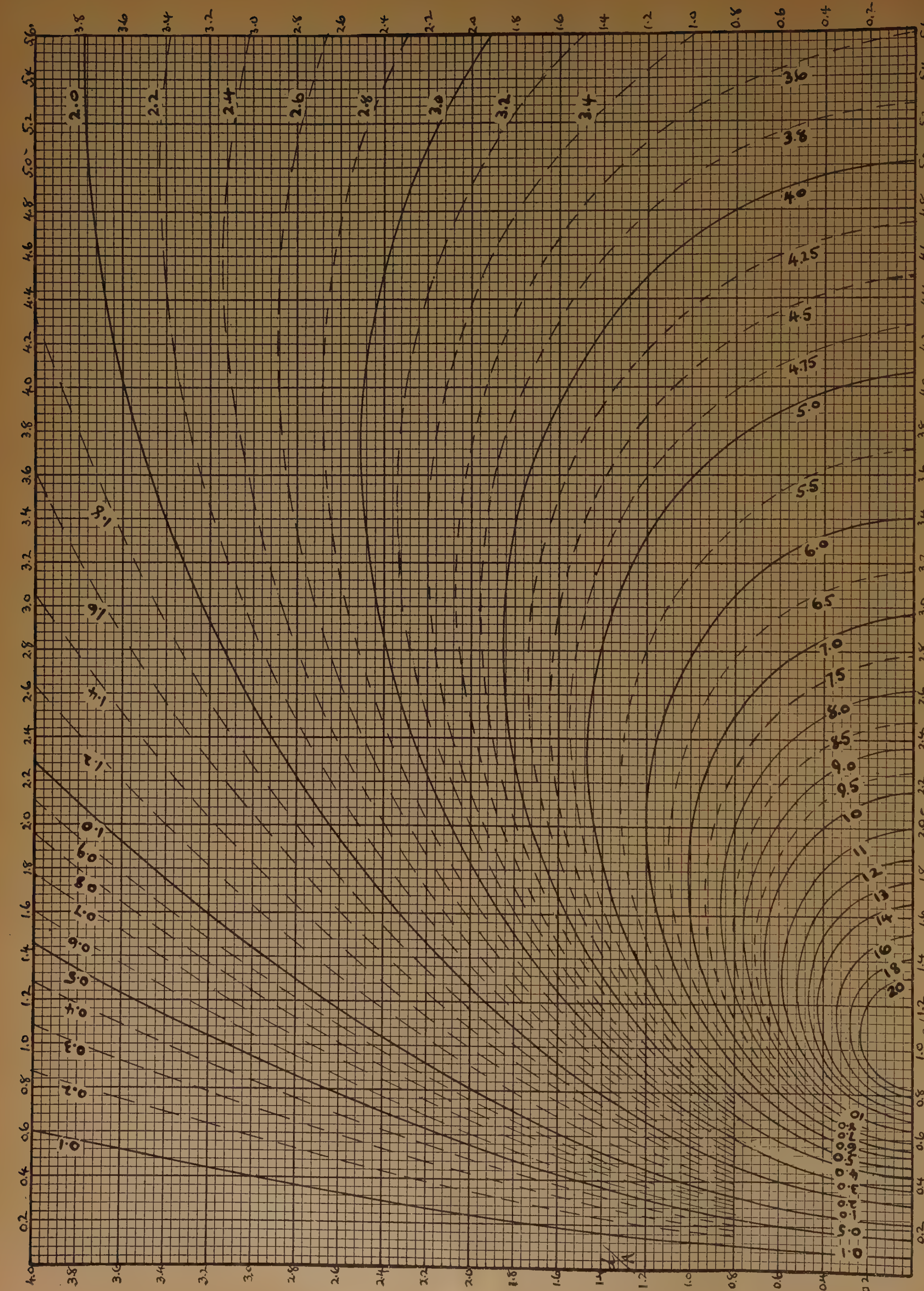


Fig. 3—Curves showing the ratio M/A of the mutual inductance of coaxial circles of the radius of the larger circle. Abscissas are ratios of the radii of the circles, a/A; ordinates are ratios of the distance d between planes and the larger radius A.

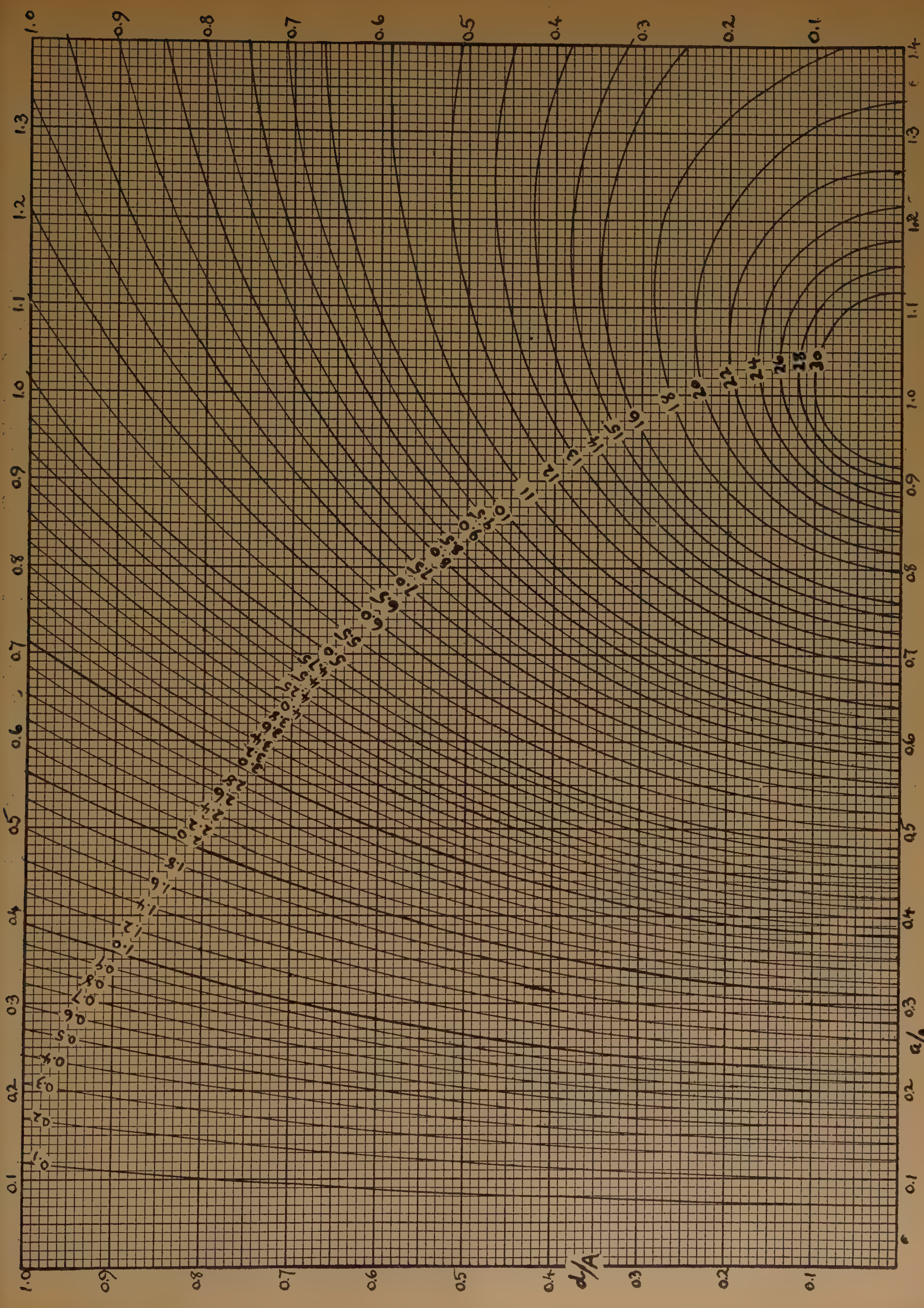


Fig. 4—Curves of M/A of coaxial circles. Detail of part of the region covered in Fig. 3 to an enlarged scale.

and the axis of Z along its axis, pass through any element $ad\phi$ of the smaller circle a circle C coaxial with the larger circle A. The radius of this circle is $a_1 = a\sqrt{1-\cos^2\phi \sin^2\theta} = aP$, and its plane is at a distance $z_1 = d + a\cos\phi\sin\theta$ from that of the circle A. The mutual inductance m of the pair of circles AC may be calculated by the formulas and tables for coaxial circles. Resolving the element $ad\phi$ of the given circle a into two components, one equal to $a_1 d\phi_1$ along the circumference of circle C and the other along its radius, the flux linkages of the latter with the field of circle A are equal to zero. Thus the flux linkages per ampere of $ad\phi$ with the circle A are the same as those of the element $a_1 d\phi_1$ of circle C with circle A. From symmetry, the mutual inductance of element $a_1 d\phi_1$ and circle A is $a_1 d\phi_1/2\pi$ times the total mutual inductance m of the circles A and C. It is found that $d\phi_1 = 1/P^2 \cos\theta d\phi$, so that (8) may be written

$$\frac{M}{A} = \frac{\cos\theta}{\pi}\int_0^\pi \frac{1}{P^2}\left(\frac{m}{A}\right)d\phi. \tag{16}$$

This expression may be calculated by mechanical quadrature by calculating values of m for coaxial circles of radii A and aP with spacing $d + a\sin\theta\cos\phi$. Referring dimensions to the larger radius A, the parameters of m are $\alpha' = \alpha P$ and $\delta' = \delta + \alpha\sin\theta\cos\phi$.

To avoid the necessity of calculating the modulus k' in order to find f so as to calculate m, the following graphical method of procedure is recommended. The accuracy attainable is a few parts in a thousand.

The curves of Figs. 3 and 4 are extensions of those given by Curtis and Sparks[6] in their paper on coaxial circles. Ordinates are values of the spacing parameter δ' and abscissas are α'. The circle of radius A has for its axis the Y axis and it passes through the points (1, 0) and (−1, 0). The curves are the loci of the co-ordinate pairs (α', δ') of circles which have the same mutual inductance with circle A. Actually, the parameters of the curves are values of m/A, that is, of the ratio of the mutual inductance in abhenries to the radius of the larger circle A in centimeters. It is evident that the curves give a picture of the flux distribution around a circular filament which cuts the axial plane (plane of the paper) in the point (1, 0) and whose axis is the Y axis. Fig. 3 is a general extended view of which Fig. 4 is one suitable for points near the circular filament.

Example 3:

As an example of the use of formula and the curves, the mutual inductance will be calculated for a pair of circles of radii $a = 10$ centimeters and $A = 20$ centimeters with the center of the smaller circle on the axis of the larger and with a distance between centers of 20 centimeters. The axes will be assumed to have an inclination of 30 degrees.

Here

$a = 10,\quad A = 20,\quad d = 20$

$\alpha = 1/2,\quad \delta = 1,\quad \theta = 30$ degrees, $\quad \sin\theta = 1/2$

$P^2 = 1 - (1/4)\cos^2\phi,\quad \delta' = 1 + (1/4)\cos\phi.$

The calculation of the ordinates used in the mechanical quadrature may then be arranged as shown in Table V. For each point (α', δ') the value of m/A is obtained

TABLE V

ϕ Degrees	$(1/4)\cos^2\phi$	P^2	$\alpha' = \alpha P$	$(1/4)\cos\phi$	$\delta' = 1 + (1/4)\cos\phi$	$\frac{m}{A}$	Ordinate $1/P^2(m/A) = y_n$
0	0.2500	0.7500	0.4330	0.2500	1.2500	0.89	1.187
15	0.2333	0.7667	0.4378	0.2415	1.2415	0.90	1.174
30	0.1875	0.8125	0.4507	0.2165	1.2165	0.95	1.169
45	0.1250	0.8750	0.4677	0.1768	1.1768	1.10	1.257
60	0.0625	0.9375	0.4841	0.1250	1.1250	1.26	1.344
75	0.0167	0.9833	0.4958	0.0647	1.0647	1.45	1.475
90	0	1	0.5000	0	1.0000	1.60	1.600
105	0.0167	0.9833	0.4958	−0.0647	0.9353	1.77	1.800
120	0.0625	0.9375	0.4841	−0.1250	0.8750	1.86	1.984
135	0.1250	0.8750	0.4677	−0.1768	0.8232	1.89	2.160
150	0.1875	0.8125	0.4507	−0.2165	0.7835	1.88	2.314
165	0.2333	0.7667	0.4378	−0.2415	0.7585	1.85	2.413
180	0.2500	0.7500	0.4330	−0.2500	0.7500	1.83	2.440

by interpolation between the curves. Denoting the ordinates as $y_0, y_1, y_2, \cdots, y_{12}$, Simpson's formula gives for the integral the formula

$$S = 1/3(\pi/12)[2(y_0 + y_2 + y_4 + \cdots + y_{12}) + 4(y_1 + y_3 + y_5 + \cdots + y_{11}) - (y_0 + y_{12})] \tag{17}$$

and the corresponding formula yielded by the Weddle rule is

$$W = 3/10(\pi/12)[5(y_1 + y_5 + y_7 + y_{11}) + 6(y_3 + y_9) + (y_0 + y_2 + y_4 + \cdots + y_{12}) + y_6]. \tag{18}$$

Applying these formulas to the calculated ordinates the numerical work may be systematized in some such manner as shown in Table VI.

TABLE VI

y_{2n}	y_{2n+1}	y_0+y_{12}	$y_1=1.174$	$5\times 6.862=34.310$
1.187	1.174	1.187	$y_5=1.475$	$6\times 3.417=20.502$
1.169	1.257	2.440	$y_7=1.800$	$\Sigma y_{2n}=12.038$
1.344	1.475		$y_{11}=2.413$	$y_6=1.600$
1.600	1.800	3.627		
1.984	2.160		6.862	68.450
2.314	2.413			
2.440			$y_3=1.257$	
	10.279	$\frac{S}{\pi}=\frac{1}{36}(61.565)$	$y_9=2.160$	$\frac{W}{\pi}=\frac{1}{40}(68.450)$
12.038			3.417	
$24.076=2\Sigma y_{2n}$				$=1.711$
$41.116=4\Sigma y_{2n+1}$		$=1.710$		
65.192				
$3.627=y_0+y_{12}$				
61.565				

Using only six equally spaced ordinates, the value given by Simpson's rule is 1.701. Applying the correction 1/15(1.710−1.701) to the twelve point value the result is $S/\pi = 1.711$.

Therefore, $M/A = 1.711 \cos\theta = 1.482$ and $M = 20$ (1.482) = 29.64 abhenries. The value found from (4) using the tables for coaxial circles is $M/A = 1.6994 \cos\theta$.

Formulas for applying the graphical method to other cases follow.

Parallel Circles, Fig. 1:

$$\frac{M}{A} = \frac{1}{\pi}\int_0^\pi \frac{(1-(\rho/a)\cos\phi)}{V^2}\left(\frac{m}{A}\right)d\phi \tag{19}$$

where $V^2 = (1 - 2(\rho/a)\cos\phi + \rho^2/a^2)$ and the co-ordinates to be used in obtaining m/A from the curves are x = abscissas = αV and ordinates $y = \delta = d/A$.

Circles with Axes Intersecting at a Point $x=\rho$, $z=d$. Fig. 2:

The mutual inductance is given by

$$\frac{M}{A}=\frac{1}{\pi}\int_0^{\pi}\frac{(\cos\theta-(\rho/a)\cos\phi)}{Q^2}\cdot\left(\frac{m}{A}\right)d\phi \text{ abhenries} \quad (20)$$

with $Q^2=(1-\cos^2\phi\sin^2\theta-2(\rho/a)\cos\phi\cos\theta+\rho^2/a^2)$ and the curves are to be used in finding m/A graphically with the co-ordinates $x=\alpha Q$ and $y=\zeta=\delta-\alpha\sin\theta\cos\phi$. The formula to be used if the tables for coaxial circles are to be employed is

$$M=\frac{\sqrt{Aa}}{\pi}\int_0^{\pi}\frac{(\cos\theta-(\rho/a)\cos\phi)}{Q^{3/2}}fd\phi \text{ microhenries} \quad (21)$$

with the argument

$$k'^2=\frac{(1-\alpha Q)^2+\zeta^2}{(1+\alpha Q)^2+\zeta^2}$$

Most General Case. Fig. 5:

Take the origin at the center of the larger circle, with the axis of Z along the axis of this circle. The XZ plane is passed through the center O' of the smaller circle of radius a. The co-ordinates of O' are $x=\rho$, $y=0$, $z=d$. To orient the axis of the circle of radius a, imagine a sphere taken with O' as center. The trace P of this axis is given by the longitude ψ, reckoned clockwise from the XZ plane and a colatitude θ, reckoned from the axis $O'Z'$ taken parallel with OZ.

The general formula for the graphical method is found to be

$$\frac{M}{A}=\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{m}{A}\right)\frac{[\cos\theta-(\rho/a)(\cos\psi\cos\phi-\sin\psi\sin\phi\cos\theta)]}{R^2}d\phi \text{ abhenries} \quad (22)$$

in which

$$R^2=(1-\cos^2\phi\sin^2\theta)+2(\rho/a)(\sin\psi\sin\phi-\cos\psi\cos\phi\cos\theta)+\rho^2/a^2 \quad (23)$$

and the co-ordinates used with the curves are

$$x=\alpha R,\qquad y=\delta-\alpha\sin\theta\cos\phi=\zeta.$$

If the tables for coaxial circles are to be used, the expression is

$$M=\frac{\sqrt{Aa}}{2\pi}\int_0^{2\pi}\frac{[\cos\theta-(\rho/a)(\cos\psi\cos\phi-\sin\psi\sin\phi\cos\theta)]}{R^{3/2}}f\,d\phi \text{ microhenries} \quad (24)$$

and f is to be taken from the tables with the argument

$$k'^2=\frac{(1-\alpha R)^2+\zeta^2}{(1+\alpha R)^2+\zeta^2}. \quad (25)$$

The general case includes in additon to the special cases already considered, two others; namely,

$\psi=90$ degrees (inclined axes but with the axes in parallel planes, separated by a distance ρ.)

$\theta=90$ degrees (axis of one circle in a plane perpendicular to the axis of the other).

For these cases the general formula is readily modified to take these conditions into account.

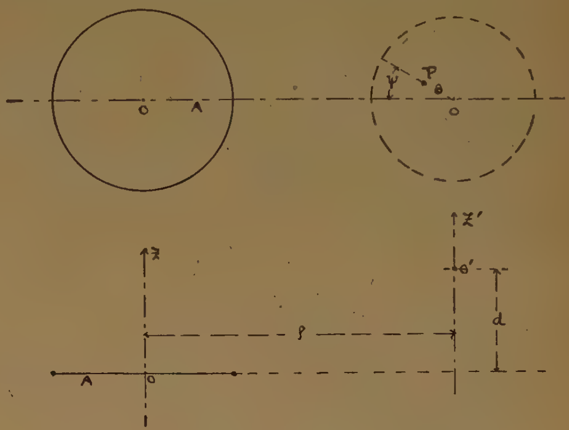


Fig. 5—Co-ordinates of circular filaments placed in any desired positions.

All these formulas apply, strictly, only to circular filaments, and accurately enough to coils whose cross-sectional dimensions are negligible, compared with the other dimensions. For coils of appreciable cross sections, a first approximation is obtained by calculating the mutual inductance of the filaments passing through the centers of the cross sections and multiplying this value by the product of the numbers of turns in the two coils. A better approximation could be obtained by replacing each coil by more than one filament, calculating the mutual inductances of the different pairs by the formulas of this article, and obtaining an average by some such expression as the Rayleigh quadrature formula.[13,14] In general, however, the difficulty of measuring the actual dimensions of the coils would hardly justify the labor of such a refinement.

[13] Gray, "Absolute Measurements," vol. 2, part 2, p. 403.
[14] *Bull. Bur. Stand.*, vol. 8, p. 34; 1912, or Scientific Paper 169, p. 34.

Notes on the Stability of Linear Networks*

EN-LUNG CHU†, NONMEMBER, I.R.E.

Summary—An extension of Nyquist's criterion of stability is made by imposing on the transfer factor a less stringent condition, $-\infty < \lim_{\omega \to +\infty} AJ(i\omega) = \lim_{\omega \to -\infty} AJ(i\omega) < +1$, instead of the condition, $\lim_{\omega \to \infty} |AJ(i\omega)| = 0(1/\omega)$. With this extension, the criterion will be found more convenient for investigating those networks, such as those with lumped constants, which very often do not satisfy the original condition. A rule, which was given intuitively by Llewellyn for extending Nyquist's criterion to more general cases, is verified. It is applicable to both unilateral and bilateral systems. In this rule, the impedance or admittance function of the circuit plays the same role as the function of $1-AJ(i\omega)$ in Nyquist's criterion. Application of the rule to impulsive networks is also considered.

Foreword*

Stability is often spoken of in connection with electrical-circuit networks. It is not entirely easy to define stability in precise terms, though the intuitional concept is fairly clear. We usually mean that a circuit is stable when it does not oscillate. Sometimes, however, the absence of runaway transients is intended. For the present paper the scope of discussion will be confined to an electrical network in which the current-voltage relations may be expressed by linear differential equations. This restriction is broad enough to include the presence of amplifiers, but these amplifiers have to be idealized to the extent that they comply with the linear restriction expressed above. This point is very important, for it immediately rules out of consideration any sudden closing of switches which apply the plate and screen biasing potentials to a vacuum tube, for the static characteristics of the tube are not linear over the wide range of voltage variation which results.

Therefore, in defining stability, the initial conditions have to be chosen in such a way that the system is "stable" before a change of some kind is made. The change is then made and it is required to determine whether the system thereafter remains stable. It is at once evident that a good deal of caution, and perhaps some intuitional ideas, have to be observed in determining how to make this change and at the same time insure that the linear restriction is not violated at the outset. For example, it might be stipulated that a switch should be closed in a portion of the network where its closure would not alter the direct currents flowing in the various meshes. This sounds simple, but actually is not, as may be seen by considering what would happen if the switch were placed in series with a condenser, and the combination were shunted across two points in the network where there previously existed a constant potential difference of the order of a hundred volts or more. The sudden closing of such a switch would be expected to produce large enough changes to carry the system well beyond the linear region of operation of most vacuum tubes.

In this particular example the variations may be limited by providing that the switch shall short-circuit a coil, and hence shall not produce any major change in the operating conditions. Besides going only part way, such a limitation imposes greater restrictions on the generality of the analysis than seem to be desirable or necessary, and leads to the conclusion that a certain amount of idealization may be required in order to define the objective.

In mathematical language, the idealization may be stated in much simpler terms. It is merely this: Given a network which can be described by a system of linear differential equations. It is desired to find a simple rule for determining whether, in the absence of applied forces (voltages or currents), the solution or solutions of these equations contain any term which, with increasing time, does not approach zero as a limit.

Stated in this way, the difficulties concerning the closing of switches is eliminated from the mathematical portion of the problem, and relegated to the physical. This makes the application no less difficult because the clause "which can be described by a system of linear differential equations" now carries the burden of intuitional concept. At least the separation between straightforward analysis and experimental realization of the conditions involved is clean-cut. The present paper, then, will deal with the mathematical problem, and will show how the application can be achieved in certain particular cases, where it is fairly evident that the network can be described by a system of linear differential equations.

The history of the problem really goes back to Cauchy who showed that, if in a region having a given boundary C the only critical points of the one-valued function $f(z)$ are poles interior to C, then

$$\int_C \frac{f'(z)}{f(z)} dz = 2\pi i(M - N)$$

where M is the number of zeros and N the number of poles within C, each point being taken a number of times equal to its order.

By writing

$$f(z) = Ae^{i\theta}$$

where A and θ are real numbers, it follows immediately that

$$\int_C d\theta = M - N$$

and hence that the difference between the number of roots and poles within C is equal to the number of times that a polar graph of $f(z)$ encircles the origin as z moves

* Decimal classification: R142. Original manuscript received by the Institute, March 24, 1944; revised manuscript received, May 17, 1944. Because of the difficulty of communicating with Dr. Chu, and to provide the reader with appropriate background material for his paper, this Foreword has been extended beyond the original manuscript by minor editing.

† Institute of Physics, Academia Sinica, Kweilin, China.

around the boundary C. When the boundary C is selected so that it encloses no poles, then the number of revolutions around the origin equals the number of zeros enclosed.

Routh applied this theorem[1] to determine the stability of linear systems. He did this by choosing the boundary C to include the portion of the complex plane of z in which the real part of z was positive. However, he seems not fully to have appreciated the simplicity of the theorem as stated above, but went to the trouble of counting the number of times the polar graph of $f(z)$ crossed the axis from $(-)$ to $(+)$ and from $(+)$ to $(-)$.

In 1875 Hurwitz[2] attacked the problem and stated the rule for stability criterion in the form of an algebraic theorem giving the necessary and sufficient conditions, under which the determinantal equation of a network can have only roots with negative real parts. His rule can be applied to any linear circuits whatever, unilateral or bilateral, provided the number of degrees of freedom is finite. The process, however, may become quite involved, if not impractical, when the number of degrees of freedom is large.

In 1932 Nyquist[3] stated a criterion which can be applied to continuous systems, as well as those with a finite number of degrees of freedom, but is limited by the following restrictions:

1. The system is unilateral.
2. The transfer factor, by which the amplifier and feedback circuit modify the current in one round trip, approaches zero, when the frequency approaches infinity. Both the Hurwitz and the Nyquist criteria have only limited fields of appreciation, and most likely the Hurwitz criterion does not admit an easy extension. Therefore, it is desirable to extend Nyquist's criterion to more general cases.

Introduction

In a previous paper,[4] extension of Nyquist's criterion to bilateral circuits has been discussed. Several methods are feasible but each has certain limitations. One of general interest is that given by Llewellyn.[5] He took the rule on faith with neither proof nor the proper restrictions. His rule is as follows:

If an R-X diagram, which in general may include negative as well as positive frequencies, encircles the origin in a clockwise direction, then the system represented by the diagram will oscillate when the terminals across which the impedance was measured are connected together.

In this paper, we shall first show that the second restriction of Nyquist's criterion is not quite necessary. A direct extension of the criterion will be made by replacing it with a less stringent condition. The original one, though not of consequence from a physical standpoint, is often inconvenient when networks with lumped constants are considered, because it is often not satisfied by such networks. Following this, we shall discuss the fundamental properties of the impedance and admittance functions of linear networks. The results of the discussion enable us to formulate the rule given by Llewellyn with certain restrictions not at all stringent. It is the one most general and simple extension of Nyquist's criterion from which other methods of extension can be easily justified. Last, we shall conclude with the application of the rule to impulsive networks.

Extension of Nyquist's Criterion to the Case Where $-\infty < \lim\limits_{\omega\to+\infty} AJ(i\omega) = \lim\limits_{\omega\to-\infty} AJ(i\omega) < +1$

Let us consider the circuit made up of an amplifier in tandem with a network. The amplifier is characterized by the amplifying ratio A which is independent of frequency. The network is characterized by the ratio $J(i\omega)$ which is a function of frequency but does not depend on the gain. The total effect of the amplifier and the network is to multiply the wave by the ratio $AJ(i\omega)$. The circuit is stable when it is broken and terminated with iterative impedances.

Suppose that a current or voltage wave e^{-bt} (b a small positive number), which dies out eventually, is introduced into the input mesh of the system at time $t=0$. Let the response in the output mesh be denoted by $AG(t)$. $AG(t)$ is to satisfy the following restrictions:

$$G(t) \text{ has bounded variation}, \quad -\infty < t < \infty. \tag{GI}$$

$$G(t) = 0, \quad -\infty < t < 0. \tag{GII}$$

$$\int_{-\infty}^{\infty} |G(t)|\, dt \text{ exists.} \tag{GIII}$$

By means of Fourier integrals we may express $AG(t)$ by the equation

$$AG(t) = \frac{1}{2\pi i}\int_I \frac{AJ(i\omega)}{i\omega + b} e^{i\omega t} d(i\omega), \tag{1}$$

where

$$\frac{AJ(i\omega)}{i\omega + b} = \int_{-\infty}^{\infty} AG(t)e^{-i\omega t}dt. \tag{2}$$

Now we define the functions

$$\frac{w(z)}{z+b} = \frac{1}{2\pi i}\int_I \frac{AJ(i\omega)}{(i\omega+b)(i\omega-z)} d(i\omega), \quad 0<x<\infty, \tag{3}$$

and

$$\frac{n(z)}{z+b} = \frac{1}{2\pi i}\int_I \frac{AJ(i\omega)}{(i\omega+b)(i\omega-z)} d(i\omega), \quad -\infty<x<0 \tag{4}$$

where

$$z = x + iy, \tag{5}$$

and where x and y are real. We define further

$$\frac{w(iy)}{iy+b} = \lim_{z\to 0} \frac{w(z)}{z+b}, \tag{6}$$

[1] E. J. Routh, "Dynamics of a System of Rigid Bodies," Macmillan and Company, Ltd., London, England, 1905. This material appears in article 291 of volume 2 of the sixth edition, published in 1905. Earlier editions, of which the first was published in 1860, may also have included it. Most libraries do not have these earlier editions.

[2] A. Hurwitz, *Mathematische Annal.*, vol. 46, pp. 273; 1875.

[3] H. Nyquist, *Bell Sys. Tech. Jour.*, p. 126, 1932.

[4] E.-L. Chu, "On the stability and analysis of bilateral feedback circuits," to be published.

[5] F. B. Llewellyn, "Vacuum tube electronics at ultra-high frequencies," Proc. I.R.E., vol. 21, pp. 1532–1574; November, 1933.

and $$\frac{n(iy)}{iy+b} = \lim_{x\to 0} \frac{n(z)}{z+b}. \tag{7}$$

As shown by Nyquist, the functions $w(z)/(z+b)$ and $n(z)/z+b)$ are analytic in their respective regions of definition and at least continuous for $x=0$. In particular $n(z)=0$.

Equations (1) and (2) may then be written as

$$AG(t) = \frac{1}{2\pi i}\int_{s+} \frac{w}{z+b} e^{zt}dz, \tag{8}$$

and $$\frac{w}{z+b} = \int_{-\infty}^{\infty} AG(t)e^{-zt}dt. \tag{9}$$

In (8) the integration is along a large semicircle having origin for center and passing through the fourth and the first quadrants of the z plane. The above equations are an example of Mellin's transforms.[6]

From the restrictions on $G(t)$ [(GI) to (GIII)] and by the same process of Nyquist the following restrictions on the network may be deduced:

$$\lim_{y\to\infty} |J(iy)| = 0(1). \tag{JI}$$

$$J(iy) \text{ is continuous.} \tag{JII}$$

$$w(iy) = AJ(iy). \tag{JIII}$$

The condition (JI) is less stringent than that given by Nyquist. This is to be expected for we have assumed a wave e^{-bt} to define $G(t)$ instead of the singular function, a unit impulse.

Now we may proceed to solve the problem in a way the same as Nyquist's. Let a temporary wave $f_0(t)$ be introduced into the feedback system and an investigation is made of whether the resulting disturbance in the system dies out. $f_0(t)$ may be expressed as

$$f_0(t) = \frac{1}{2\pi i}\int_I F(z)e^{zt}dz = \frac{1}{2\pi i}\int_{s+} F(z)e^{zt}dz, \tag{10}$$

where $F(z)$ is subjected to the same restrictions as $w(z)/(z+b)$.

Summing up the original wave and the first n round trips we have a total of

$$S_n(t) = \frac{1}{2\pi i}\int_{s+} F(z)\cdot(1+w+\cdots+w^n)\cdot e^{zt}dz. \tag{11}$$

Thus the resulting current or voltage would be

$$S(t) = \frac{1}{2\pi i}\int_{s+} \frac{F(z)}{1-w} e^{zt}dz - \lim_{n\to\infty} \frac{1}{2\pi i}\int_{s+} \frac{F(z)\cdot w^{n+1}}{1-w} e^{zt}dz. \tag{12}$$

In this way, we can carry on the discussion no further, because we are unable to establish the convergency of the series. In fact, the series method of treatment is doubtful. It can not be justified by physical considerations. We have to calculate the resulting wave by a different method.

When a wave Fe^{zt} is applied at the input terminals, the response at the output terminals will be Fwe^{zt}. On closing the feedback path the same wave will have a response $s_z(t)$ such that

$$s_z(t) = Fe^{zt} + w\cdot s_z(t),$$

i.e., $$s_z(t) = \frac{F}{1-w}\cdot e^{zt}. \tag{13}$$

The response caused by the wave $f_0(t)$ will be

$$S(t) = \frac{1}{2\pi i}\int_{s+} \frac{F}{1-w}\cdot e^{zt}dz. \tag{14}$$

The integral $\int_I (F/1-w)e^{zt}dz$ exists for all values of t and approaches zero for large values of t, if $1-w$ does not equal to zero on the imaginary axis.[7]

Hence $S(t) = \dfrac{1}{2\pi i}\displaystyle\int_C \frac{F}{1-w} e^{zt}dz$ for large values of t, (15)

in which the path of integration is a closed contour first along the semicircle and then along the imaginary axis.

Since F and w are analytic functions, the integrand of the integral (15) can not have any essential singularity within the contour C. If the integrand has any poles, they must lie within a finite distance of the origin except when $\lim_{|z|\to\infty} w=1$. As will be explained later, the function w of any physically realizable network tends to a real value less than $+1$ when $|z|$ approaches infinite. Hence the integral (15) exists, because the integrand has only a finite number of poles in the contour C.

The system will be stable or unstable according to whether the number of roots of the equation $1-w=0$ within C is equal to zero or not. Remembering that the function w has no poles within C, the number of roots of the equation $1-w=0$ within the same contour will be

$$\frac{1}{2\pi i}\int_C d\log(1-w) = \frac{1}{2\pi i}\int_R d\log u. \tag{16}$$

D is a curve in the u plane which corresponds to the curve C in the z plane. If the curve D is *acyclic* with respect to the origin, the above integral will be equal to zero. Otherwise, the number of roots of the equation $1-w=0$ within C is equal to the number of times that the curve D encircles the origin. When C is taken large enough, the curve D will correspond to the path of the imaginary axis ($x=0$) in the z plane, for $\lim_{|z|\to\infty} w$ approaches a real finite value. If $w=1$ on the curve D, the above arguments can not apply, but in this case the system is certainly unstable. Hence Nyquist's rule of stability is established on a less stringent condition

$$-\infty < \lim_{y\to+\infty} AJ(iy) = \lim_{y\to-\infty} AJ(iy) < +1.$$

Plot plus and minus the imaginary part of $AJ(i\omega)$ against the real part for all frequencies from 0 to ∞. If the point $1+i0$ lies completely outside this curve the system is stable; if not, it is unstable.

For an example, let us take[8]

$$AJ(i\omega) = \frac{A(1+i\omega)}{(1+i2\omega)}. \tag{17}$$

[6] R. Courant and D. Hilbert, "Methoden der Mathematischen Physik," 1931, vol. 1, pp. 87.

[7] For the integral $\int_I F(z)\cdot e^{zt}dz$ exists for all values of t and approaches zero as t approaches infinity.

[8] See example 8 of footnote reference 3.

The diagrams of $AJ(i\omega)$ for different values of A are shown in Fig. 1. From the diagrams we may conclude that the system is stable when $A<+1$; and unstable when $+1\leq A\leq +2$. If we overlooked the restriction on $AJ(i\omega)$, we might expect that the system would be stable when $A>+2$. In this case, however, the rule does not apply. For any network which is stable when open- or short-circuited, $AJ(i\omega)$ must decrease as frequency is increased indefinitely at least to a value less than $+1$, for otherwise the network would have the characteristic of a negative resistance at infinite frequency, as will be shown in the following section. This is illustrated in Fig. 2. The point (1, 0) lies inside the locus, so the system is unstable.

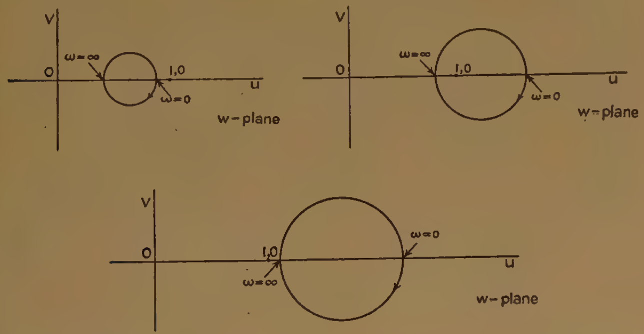


Fig. 1—Diagram of $\dfrac{A(1+i\omega)}{(1+i2\omega)}$

(a) $A<+1$.
(b) $+1<A<+2$.
(c) $A>+2$.

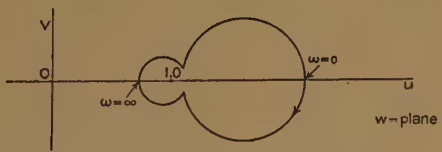


Fig. 2—Diagram of transfer factor which is equal to

$$\frac{A(1+i\omega)}{(1+i2\omega)} \qquad (A>+2)$$

when ω is finite and approaches a real value less than $+1$ when ω tends to infinity.

Discussion on the Impedance and Admittance Functions of Linear Networks

Consider a two-terminal network which is stable when open-circuited. Let the impedance function of the network be represented by $Z(i\omega)$, and its two components, resistive and reactive, by $R(\omega)$ and $X(\omega)$ respectively.

$$Z(i\omega) = R(\omega) + iX(\omega). \tag{18}$$

Let $E(t)$ denote the indicial impedance of the network, that is the response to a unit step current. $E(t)$ is to satisfy the following restrictions:

$$E'(t) \text{ has bounded variation}, \quad -\infty < t < \infty. \tag{EI}$$

$$E(t) = 0, \quad -\infty < t < 0. \tag{EII}$$

$$\int_0^\infty |E'(t)|\, dt \text{ converges}, \tag{EIII}$$

where $E'(t)=(d/dt)E(t)$. The conditions (E') and (EII) are same as (GI) and (GII) which are obviously fulfilled physically. The condition (EIII) implies that $\int_0^\infty E'(t)dt$ is convergent whether $E'(t)$ is regular or *in general* regular in the interval $(0, \infty)$. In other words, it implies that the *limited* function $E(t)$ will have none or a finite number of discontinuities and approach a finite value when $t\to\infty$, so also that

$$\int_{-\infty}^{\infty} |E(t)|\, e^{-bt}dt \text{ converges}, \tag{19}$$

where b is a small positive number. The above conditions are evidently compatible with practical experiences.

According to the superposition theorem, when a current of any wave form $i(t)$ is sent through the network the response voltage $v(t)$ across the terminals will be

$$v(t) = E(0)\cdot i(t) + \int_\epsilon^t E'(\tau)\cdot i(t-\tau)d\tau \tag{20}$$

where ϵ is an arbitrary small positive number.

Let
$$i(t) = I \sin(\omega t+\theta), \tag{21}$$

we have
$$v(t) = I\sin(\omega t+\theta)\cdot\left[E(0)+\int_\epsilon^t \cos\omega\tau\cdot E'(\tau)d\tau\right]$$
$$-I\cos(\omega t+\theta)\cdot\int_\epsilon^t \sin\omega\tau\cdot E'(\tau)d\tau. \tag{22}$$

$v(t)$ may be resolved into two parts:[9]

$$I\sin(\omega t+\theta)\cdot\left[E(0)+\int_\epsilon^\infty \cos\omega\tau\cdot E'(\tau)d\tau\right]$$
$$-I\cos(\omega t+\theta)\cdot\int_\epsilon^\infty \sin\omega\tau\cdot E'(\tau)d\tau, \tag{23}$$

which is the steady state, and

$$-I\sin(\omega t+\theta)\cdot\int_t^\infty \cos\omega\tau\cdot E'(\tau)d\tau$$
$$+I\cos(\omega t+\theta)\cdot\int_t^\infty \sin\omega\tau\cdot E'(\tau)d\tau, \tag{24}$$

which is the transient distortion, which ultimately dies away for sufficiently large values of t on account of (*EIII*).

But the steady state value of the response voltage can also be expressed as

$$I\cdot[R(\omega)\cdot\sin(\omega t+\theta)+X(\omega)\cdot\cos(\omega t+\theta)]. \tag{25}$$

By comparing with (22), we have

$$R(\omega) = E(0)+\int_\epsilon^\infty \cos\omega\tau\cdot E'(\tau)d\tau,$$

and
$$X(\omega) = -\int_\epsilon^\infty \sin\omega\tau\cdot E'(\tau)d\tau;$$

so
$$Z(i\omega) = E(0)+\int_\epsilon^\infty e^{-i\omega\tau}\cdot E'(\tau)d\tau. \tag{26}$$

From (*EIII*) it may be inferred that

$$\lim_{\omega\to\infty}\int_\epsilon^\infty e^{-i\omega\tau}\cdot E'(\tau)d\tau = 0(1).$$

[9] J. R. Carson, "Electric Circuit Theory and the Operational Calculus," 1926, p. 18.

Thus

$$\lim_{\omega \to \infty} Z(i\omega) = E(0), \tag{27}$$

which is a real quantity. $E(0)$ is certainly greater than zero, for every network will cease to function as a negative resistance when the frequency is sufficiently high.

Thus the impedance function $Z(i\omega)$ of a linear network, which is stable when open-circuited, exists at all frequencies and approaches a positive real value when $\omega \to \infty$.

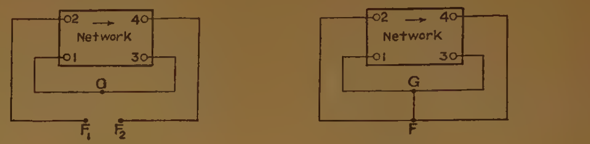
Similarly, if a linear network is stable when short-circuited, the admittance function $A(i\omega)$ will exist for all frequencies and approach a positive real value when $\omega \to \infty$.

The above method of treatment applies not only to input impedance or admittance but also to any transfer impedance or admittance or transfer factor, or any ratio of mesh voltages or currents of a stable network. All such impedances, admittances, transfer factors, or ratios must approach a finite real value when ω approaches infinity, but unlike the input impedances or admittances, they are not necessarily positive.

Now we come back to consider the network of the last section. Let a unit step current or voltage be introduced into its input mesh and $AE(t)$ denote the corresponding response in the output mesh. The restrictions on $AE(t)$ are the same as (EI) to (EIII). By the superposition theorem, we obtain the response caused by the wave e^{-bt} as

$$AG(t) = AE(0) \cdot e^{-bt} + \int_{\epsilon}^{t} AE'(\tau) \cdot e^{-b(t-\tau)} d\tau. \tag{28}$$

Since $0 < e^{-b(t-\tau)} < 1$, the integral $\int_\epsilon^t AE'(\tau) \cdot e^{-b(t-\tau)} d\tau$ is absolutely convergent for all values of t, and approaches zero for large values of t. Moreover, $AG(t)$ is a *limited* function of t for $0 < t < \infty$, and can have only a finite number of discontinuities. It can be readily shown that all the restrictions on $G(t)$ given in the last section are deducible from those[10] on $E(t)$. Besides, the integral $\int_0^\infty |G'(t)| dt$ is convergent.[11] Thus we are assured that the above reasoning applies to all linear networks which are physically realizable.



(a) (b)

Fig. 3—Feedback network.
(a) Open-circuited.
(b) Short-circuited.

With regard to the magnitude of $AE(0)$, let us find the open-circuited impedance or short-circuited admittance of the feedback network shown in Fig. 3.

$$Z_{F_1F_2}(i\omega) = [1 - AJ(i\omega)] \cdot [Z_{\text{in}}(i\omega) + Z_{\text{out}}(i\omega)]. \tag{29}$$

$$A_{FG}(i\omega) = [1 - AJ(i\omega)] \cdot \left[\frac{1}{Z_{\text{in}}(i\omega)} + \frac{1}{Z_{\text{out}}(i\omega)}\right] \tag{30}$$

Z_{in} and Z_{out} are the iterative terminating impedances.

Since the network is stable when open- or short-circuited the impedance and admittance values $Z_{F_1F_2}$, A_{FG}, Z_{in}, and Z_{out} must be real and positive when ω approaches infinity. Thus we have

$$-\infty < \lim_{\omega \to \infty} AJ(i\omega) = \lim_{\omega \to -\infty} AJ(i\omega) < +1. \tag{31}$$

On the other hand, we obtain from (9)

$$\lim_{|z| \to \infty} w(z) = \lim_{|z| \to \infty} (z) \cdot \int_0^\infty AG(t) \cdot e^{-zt} dt,$$

and by integrating by parts

$$\lim_{|z| \to \infty} w(z) = AG(0) + \lim_{|z| \to \infty} \int_0^\infty AG'(t) e^{-zt} dt. \tag{32}$$

Since the integral $\int_0^\infty |G'(t)| dt$ is convergent, we can easily prove that $\lim_{|z| \to \infty} \int_0^\infty AG'(t) e^{-zt} dt$ vanishes. Thus

$$\lim_{|z| \to \infty} w(z) = AG(0) = AE(0), \tag{33}$$

and from *(JIII)*

$$\lim_{|y| \to \infty} AJ(iy) = \lim_{|y| \to \infty} w(iy) = AE(0). \tag{34}$$

Hence

$$-\infty < AE(0) < +1. \tag{35}$$

This condition is broader than Nyquist's $(AE(0) = 0)$. It applies to all linear networks which are physically realizable, and also to those which may have lumped constants and are not physically realizable, provided $\lim_{|\omega| \to \infty} AJ(i\omega)$ is real and less than $+1$.

General Stability Criteria for Linear Networks

Having made clear the characteristics of the impedance and admittance functions of linear networks, we

---

[10] (GI) is a consequence of (EI) and (EIII). See E. C. Titchmarsh, "The Theory of Functions," 1932, p. 360. (GII) is a consequence of (EII).

$$A \cdot \int_0^\infty |G(t)| dt = A \cdot \int_0^\infty |E(0)| e^{-bt} dt + A \cdot \int_0^\infty dt \int_0^\infty |E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) d\tau$$

where $H(t-\tau)$ represents Heaviside's unit function.

The function $|E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) \geqq 0$ is regular in

$$R_{t,\tau}(0\infty, 0\infty)$$

except on certain lines $\tau = \tau_1, \tau_2, \cdots$. The inetgral

$$\int_0^\infty |E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) d\tau$$

is uniformly convergent in $R_t(0, \infty)$, for it exists for any value of t. See J. Pierpont, "The Theory of Functions of Real Variables," 1905, vol. 1, p. 426. The integral

$$\int_0^\infty |E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) dt$$

is uniformly convergent in $R_\tau(0, \infty)$ except at $\tau_1, \tau_2, \cdots$.

Since we know that

$$\int_0^\infty d\tau \int_0^\infty |E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) dt$$

is convergent, so also

$$\int_0^\infty dt \int_0^\infty |E'(\tau)| \cdot e^{-b(t-\tau)} \cdot H(t-\tau) d\tau$$

is convergent. (J. Pierpont, loc. cit., p. 489.) Hence the integral $\int_0^\infty |G(t)| dt$ is convergent. This is (GIII).

[11] On differentiating (28), we have

$$G'(t) = -b \cdot E(0) \cdot e^{-bt} + E'(t) - b \cdot \int_\epsilon^t E'(\tau) \cdot e^{-b(t-\tau)} d\tau.$$

From footnote 10, it is obvious that the integral $\int_0^\infty |G'(t)| dt$ exists.

may extend Nyquist's regeneration theory to bilateral systems. We consider two-terminal networks, unilateral or bilateral, which belong to either of the following two classes:

1. Networks which are stable when open-circuited.
2. Networks which are stable when short-circuited.

Case 1—Stable Open-Circuited Networks

As shown in the above section, the impedance function of the network is the sum of two terms, one being a constant and the other approaching zero as $\omega \to \infty$.

$$Z(i\omega) = E(0) + \int_{\epsilon}^{\infty} e^{-i\omega\tau} \cdot E'(\tau) d\tau$$
$$= R(\infty) + Z_0(i\omega). \quad (36)$$

From the restrictions on $E(t)$ or $G(t)$, those following on $Z(i\omega)$ may be deduced:

$$\lim_{y\to\infty} |Z_0(iy)| = 0(1) \quad \text{(ZI)}$$
$$Z_0(iy) \text{ is continuous,} \quad \text{(ZII)}$$
$$\lim_{x\to 0} w_0(z) = Z_0(iy), \quad \text{(ZIII)}$$

where

$$\frac{w_0(z)}{z+b} = \frac{1}{2\pi i}\int_I \frac{Z_0(i\omega)}{(i\omega+b)(i\omega-z)} d(i\omega), \quad 0<x<\infty. \quad (37)$$

Suppose that the network is short-circuited and a temporary wave $f_0(t)$ is introduced into it and we wish to determine whether the resultant disturbance dies out. Let the operational expression of $f_0(t)$ be $F(z)$ which is defined as before and is subjected to the same restrictions as $w_0(z)$ except that $\lim_{y\to\infty} |F(iy)| = 0(1/y)$. $F(z)$ has no poles with $x \geqq 0$. The response current in the network is

$$i(t) = \frac{1}{2\pi i}\int_{s+} \frac{F(z)}{R(\infty) + w_0(z)} e^{zt} dz$$
$$= \frac{1}{2\pi i}\int_{s+} \frac{F(z)}{w(z)} e^{zt} dz, \quad (38)$$

where $w(z)$ is an analytic function for $0<x<\infty$ and at least continuous for $x=0$.

For large values of t, $\int_I [F(z)/w(z)]e^{zt}dz$ approaches zero if $w(z)$ does not equal to zero on the imaginary axis. Thus

$$i(t) = \frac{1}{2\pi i}\int_C \frac{F(z)}{w(z)} e^{zt} dz \text{ for large values of } t. \quad (39)$$

The system will be stable if $i(t) \to 0$ as $t \to \infty$, otherwise it is unstable.

By the same arguments as used by Nyquist, we have the following rule:

Let $Z(i\omega)$ be the impedance function of a two-terminal network which is stable when *open-circuited.* Plot plus and minus the imaginary part of $Z(i\omega)$ or $1/Z(i\omega)$ against the real part for all frequencies from 0 to ∞. If the origin lies completely outside this curve, the system when *short-circuited* will be stable; if not, it is unstable.

Case 2—Stable Short-Circuited Networks

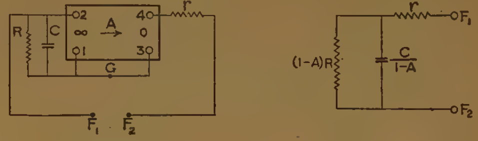
In a similar way, we may enunciate the following rule:

Let $A(i\omega)$ be the admittance function of a two-terminal network which is stable when short-circuited. Plot plus and minus the imaginary part of $A(i\omega)$ or $1/A(i\omega)$ against the real part for all frequencies from 0 to ∞. If the origin lies completely outside this curve, the system which open-circuited will be stable; if not, it is unstable.

The applications of the above criteria are illustrated by the following examples:

1—Series-type feedback circuit.

The schematic connection diagram and the equivalent circuit diagram of a series-type feedback circuit are



(a) (b)

Fig. 4—Series-type feedback circuit.
(a) Schematic-connection diagram. (b) Equivalent-circuit diagram.

shown in Fig. 4(a) and Fig. 4(b) respectively. The circuit is stable when open-circuited and

$$Z_{F_1F_2}(i\omega) = r + \frac{(1-A)R}{1+(\omega CR)^2} - i\frac{(1-A)\omega CR^2}{1+(\omega CR)^2}. \quad (40)$$

Let $A>1$. The $R-X$ diagrams of $Z_{F_1F_2}(i\omega)$ are shown in Fig. 5. If $r<(A-1)R$, the diagram is a circle enclosing

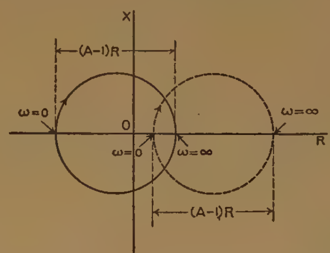


Fig. 5—$R-X$ diagrams of a series-type feedback circuit. Solid curve for $r<(A-1)R$; dotted curve for $r>(A-1)R$. $A>1$.

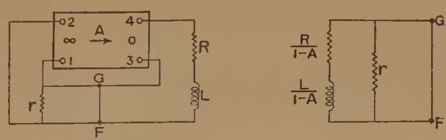
the origin; so the system will be unstable when the terminals F_1, F_2 are short-circuited. If $r>(A-1)R$, the same system will be stable.

2—Shunt-type feedback circuit.

The schematic connection diagram and the equivalent circuit diagram are shown in Fig. 6(a) and Fig. 6(b) respectively. The system is stable when short-circuited, and

$$A_{FG}(i\omega) = \frac{1}{r} + \frac{(1-A)R}{R^2+(\omega L)^2} - i\frac{(1-A)\omega L}{R^2+(\omega L)^2}. \quad (41)$$

Let $A>1$. The G–B diagrams of $A_{FG}(i\omega)$ are shown in Fig. 7. If $(A-1)r \geq R$, the system will be unstable when



(a) (b)

Fig. 6
(a) Schematic-connection diagram. Shunt-type feedback circuit.
(b) Equivalent-circuit diagram.

the shunt path across FG is open-circuited. If $(A-1)r<R$, the same system will be stable.

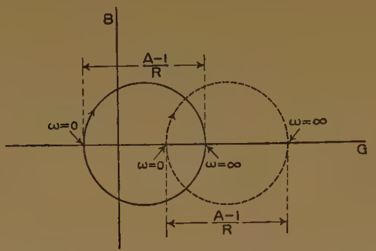


Fig. 7—G–B diagrams of a shunt-type feedback circuit. Solid curve for $(A-1)r>R$; dotted curve for $(A-1)r<R$. $A>1$.

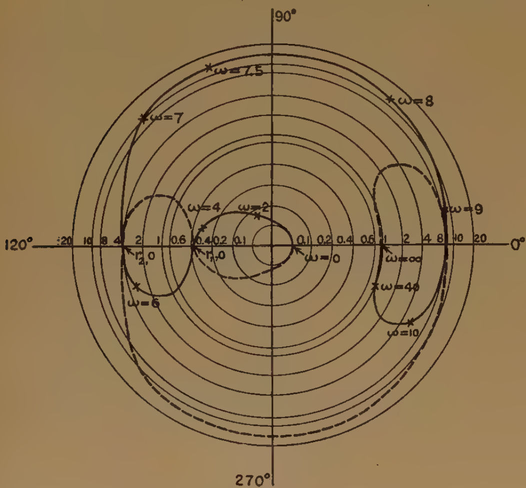


Fig. 8—$R-X$ diagram of the function
$$\frac{[4+i(\omega-2)]\cdot[4+i(\omega+2)]\cdot[2+i(\omega-2)]\cdot[2+i(\omega+2)]}{[1+i(\omega-8)]\cdot[1+i(\omega+8)]\cdot[1+i(\omega-7)]\cdot[1+i(\omega+7)]}$$
on logarithmic amplitude scale.

3—Impedance function with specified zeros and poles. Let us take[12]

$$Z(i\omega)=r+\frac{[4+i(\omega-2)]\cdot[4+i(\omega+2)]\cdot[2+i(\omega-2)]\cdot[2+i(\omega+2)]}{[1+i(\omega-8)]\cdot[1+i(\omega+8)]\cdot[1+i(\omega-7)]\cdot[1+i(\omega+7)]}. \tag{42}$$

The circuit is stable when open-circuited, for $Z(p)$ has no poles with positive real part. The R–X diagram of $Z(i\omega)$ for $r=0$ is shown in Fig. 8. Although the locus cuts the negative real axis at two points, it does not encircle the origin. This is to be expected for $Z(p)$ has no zeros with positive real part for $r=0$. The circuit is also stable when short-circuited.

Let $r>0$ and be increased gradually to a large value, the curve will move bodily towards right. When $|r_1|\leq r\leq |r_2|$, the system when short-circuited will become unstable. When $r>|r_2|$, the same system will again be stable.

Impulsive Networks

Impulsive networks, though not physically realizable, are often met with in practical calculations when certain approximations are made (e.g. when lumped constants are assumed). The impedance or admittance functions of such networks may approach either zero or infinity when frequency approaches infinity. It is obvious that we may avoid the difficulty by introducing into the network the proper resistance which is always actually present in a mesh or between two junctions. However, it is sometimes convenient to modify the rule such that we may consider impulsive networks also.

As before, we consider stable open-circuited networks first. In case $\lim_{\omega\to\infty}|Z(i\omega)|\to\infty$, we have

$$Z(i\omega)=\overline{E}(0)+\int_\epsilon^\infty e^{-i\omega\tau}\cdot\overline{E}'(\tau)\cdot d\tau+i\omega\overline{L}, \tag{43}$$

where $\overline{E}(\tau)$ is the finite portion of the response to a unit step current applied at $t=0$, and $\overline{L}$ is the impulsive inductance of the network. The total response voltage caused by the unit step current is

$$E(\tau)=\dot{\overline{E}}(\tau)+\lim_{a\to0}\frac{\overline{L}}{a}\cdot e^{-\pi\tau^2/a^2}. \tag{44}$$

The expression $\lim_{a\to0}(1/a)e^{-\pi\tau^2/a^2}$ represents the singular function of a unit impulse which is zero everywhere except at $\tau=0$. Thus

$$w(z)=\overline{L}(z)+\overline{E}(0)+w_0(z), \tag{45}$$

where $w_0(z)$ has the same definition of (37).

When the network is closed and a temporary voltage wave $f_0(t)$ is introduced into its input mesh, the response current will be

$$i(t)=\frac{1}{2\pi i}\int_{s+}\frac{F(z)}{\overline{L}(z)+\overline{E}(0)+w_0(z)}\cdot e^{zt}dz. \tag{46}$$

Here $F(z)$ has only to satisfy the condition that $\lim_{y\to\infty}|F(iy)|=0(1)$. For large values of t, the integral (46) can be written as

$$\frac{1}{2\pi i}\int_C\frac{F(z)}{w(z)}\cdot e^{zt}dz.$$

The question of stability may be decided by finding the number of roots of the equation $w(z)\infty0$. Now since $\lim_{|z|\to\infty}w(z)$ does not exist, Cauchy's theorem cannot be applied. Instead, we may consider the equation $w(z)/z+r=0$ (r being any positive real value), which as the equation $w(z)=0$ has no poles in the region $0<x<\infty$. The function $Z(i\omega)/i\omega+r$ is the ratio of the impedance of the network to that of an inductive element having series resistance r and inductance unity. Thus the rule enunciated in the above section applies if we replace $Z(i\omega)$ by the ratio $Z(i\omega)/i\omega+r$. $\lim_{\omega\to\infty}Z(i\omega)/iw+r$ is real and positive.

In case $\lim_{\omega\to\infty}Z(i\omega)=0\ (1/\omega)$, we have

$$Z(i\omega)=\int_\epsilon^\infty e^{-i\omega\tau}\cdot E'(\tau)d\tau \tag{47}$$

and

$$i(t)=\frac{1}{2\pi i}\int_{s+}\frac{F(z)}{w_0(z)}\cdot e^{zt}dz. \tag{48}$$

Here $F(z)$ has to satisfy the condition that

[12] D. G. Reid, *Wireless Eng.*, vol. 14, p. 588; 1937.

$$\lim_{y\to\infty}\left|\frac{F(iy)}{w_0(iy)}\right| = 0\left(\frac{1}{y}\right).$$

For large values of t, the integral (48) is equal to

$$\frac{1}{2\pi i}\int_C \frac{F(z)}{w_0(z)}\cdot e^{zt}dz.$$

As before the stability of the system is determined by finding the number of roots of the equation $w(z)=w_0(z)=0$. In this instance, we consider the equation $w(z)\cdot(Rz+1)/R=0$, R being any positive real value. The function $Z(i\omega)\cdot(Ri\omega+1)/R$ is the ratio of the impedance of the network to that of a capacitive element having shunt resistance R and capacitance unity. We may apply the rule by using the ratio function in place of $Z(i\omega)$.

$$\lim_{\omega\to\infty} Z(i\omega)\cdot\frac{Ri\omega+1}{R} \text{ is real and positive.}$$

The case of stable short-circuited networks may be treated similarly.

Conclusions

1. Nyquist's criterion of stability has been extended to the case $-\infty < \lim_{|\omega|\to\infty} AJ(i\omega) < +1$.

2. The impedance function of any physically realizable linear network which is stable when open-circuited is finite for all frequencies from 0 to ∞. It approaches a positive real value as $\omega\to\infty$.

3. The same is true with the admittance function of any physically realizable linear network which is stable when short-circuited.

4. Llewellyn's rule for generalizing Nyquist's criterion has been verified. It is applicable to any linear network, unilateral or bilateral, which is stable either in open-circuited or in short-circuited condition.

5. The rule with slight modification is applicable to impulsive networks which do not satisfy the conditions stated in 2 and 3. In applying the rule it is important to observe that the impedance or admittance function must be finite for all finite values of frequency.

Navy Honors RCA Laboratories

For development of radio devices, which "at first checked and then started the enemy down the road to total defeat," RCA Laboratories at Princeton, N. J., recently were honored by the Industrial Incentive Division of the United States Navy, in coöperation with WCAU, Philadelphia, in the broadcast of "A Salute to Uncle Sam's Industries," dedicated to men and women on the production line.

Guests of the Navy and WCAU in the salute ceremony in honor of "RCA's notable achievements" and "invaluable contributions both to the prosecution of the war and to life in the postwar world," included E. W. Engstrom, Fellow of the IRE, research director of the RCA Laboratories, and his associates, and W. D. Hershberger, Associate Member of the Institute, active in the field of short-wave radio and radar for more than 10 years.

Effects of wartime research on radio in the future will provide better systems and sharpen tools and techniques, Mr. Engstrom said in the broadcast, while a tremendous amount of knowledge about short waves will have been developed.

"We were pressed into this by the war emergency," he stated, "and therefore work was telescoped which would normally have taken years to accomplish. The war has provided field experience for testing equipment which the laboratories had been developing, in association with the Army and Navy. After the war, the use of this equipment will be expanded to all means of transportation. Radio and electronics, through their broad realm of application, will make it possible for planes, ships, and motor vehicles to come and go in safety without regard to weather, obstacles, or the possibility of collision. The tracks for their movements will be the 'rails and channels' of radio."

"Electronics also will assume new roles," Mr. Engstrom continued. "We shall count and compute by electronics. We shall see by visible and invisible light. Generators of radio frequency will take their places in the factory, in the field, and even in the home, to heat and to effect various reactions and changes. I am convinced that radio, electronics, and acoustics will have an important part in science, in culture, and in just ordinary living, when the war is won."

The advantage of short-wave radio in opening up more channels of communication was described by Dr. Hershberger, who flew with what was probably the first airborne radar equipment in the world. "As you go into shorter and shorter wavelengths, you can confine radio signals to narrower avenues," he explained. "Thus, you can have signals radiating in all directions which do not interfere with one another. We are learning more about short waves all the time, and there is a great deal of unexplored territory in the field of short-wave radio.

Dr. Hershberger expressed the opinion that when the full history of the development of radar can be told that the public will be astounded at the speed with which it was brought to a state of usefulness. He predicted that the detection services which have been developed during the war will have a powerful effect upon civilian flying in peacetime by letting airplanes know their distance from the ground, the location of mountains and other obstacles, and permit flying without regard to weather.

THE INSTITUTE OF RADIO ENGINEERS

INCORPORATED

1945 WINTER TECHNICAL MEETING
NEW YORK, N.Y.—JANUARY 24, 25, 26, AND 27, 1945

SECTION MEETINGS

ATLANTA October 20	CHICAGO October 20	CLEVELAND October 26	DETROIT October 20	LOS ANGELES October 16
NEW YORK November 1	PHILADELPHIA November 2	PITTSBURGH November 13	PORTLAND November 13	WASHINGTON November 13

SECTIONS

ATLANTA—Chairman, Walter Van Nostrand; Secretary, Ivan Miles, 554—14 St., N. W., Atlanta, Ga.

BALTIMORE—Chairman, W. I. Webb; Secretary, H. L. Spencer, Box 6760, Towson 4, Md.

BOSTON—Chairman, Charles C. Harris; Secretary, Corwin Crosby, 16 Chauncy St., Cambridge, Mass.

BUENOS AIRES—Chairman, L. C. Simpson; Secretary, I. C. Grant, Venezuela 613, Buenos Aires, Argentina.

BUFFALO-NIAGARA—Chairman, A. J. Dybowski; Secretary, H. G. Korts, 51 Kinsey Ave., Kenmore, N. Y.

CHICAGO—Chairman, W. O. Swinyard; Secretary, A. W. Graf, 135 S. LaSalle, St., Chicago 3, Ill.

CINCINNATI—Chairman, J. L. Hollis; Secretary, R. S. Butts, Box 1403, Cincinnati 2, Ohio.

CLEVELAND—Chairman, A. S. Nace; Secretary, Lester L. Stoffel, 1095 Kenneth Dr., Lakewood, Ohio

CONNECTICUT VALLEY—Chairman, R. E. Moe; Secretary, L. A. Reilly, 989 Roosevelt Ave., Springfield, Mass.

DALLAS-FORT WORTH—Chairman, D. J. Tucker; Secretary, P. C. Barnes, WFAA-WBAP, Grapevine, Texas.

DAYTON— Chairman, George L. Haller; Secretary, Joseph General, 1319 Superior Ave., Dayton 7, Ohio.

DETROIT—Chairman, R. A. Powers; Secretary, R. R. Barnes, 1411 Harvard Ave., Berkley, Mich.

EMPORIUM—Chairman, H. D. Johnson; Secretary, A. Dolnick, Sylvania Electric Products, Inc., Emporium, Pa.

INDIANAPOLIS—Chairman, H. I. Metz; Secretary, J. D. Colvin, 328 E. 47 St., Indianapolis, Ind.

KANSAS CITY—Chairman, A. P. Stuhrman; Secretary, R. N. White, 4800 Jefferson St., Kansas City, Mo.

LOS ANGELES—Chairman, L. W. Howard; Secretary, Frederick Ireland, 1000 N. Seward St., Hollywood 38, Calif.

MONTREAL—Chairman, F. S. Howes; Secretary, J. A. Campbell, Northern Electric Co., Ltd., 1261 Shearer St., Montreal, Que., Canada.

NEW YORK—Chairman, Lloyd Espenschied; Secretary, J. E. Shepherd, 111 Courtenay Rd., Hempstead, L. I., N. Y.

PHILADELPHIA—Chairman, T. A. Smith; Secretary, S. Gubin, RCA Victor Division, Radio Corporation of America Bldg. 8–10, Camden, N. J.

PITTSBURGH—Chairman, T. C. Kenny; Secretary, R. K. Crooks, Box 2038, Pittsburgh 30, Pa.

PORTLAND—Chairman, W. A. Cutting; Secretary, W. E. Richardson, 5960 S.W. Brugger, Portland, Ore.

ROCHESTER—Chairman, G. R. Town; Secretary, Harold Goldberg, Stromberg-Carlson Co., Rochester 3, N. Y.

ST. LOUIS—Chairman, N. B. Fowler; Secretary, C. H. Meyer, KFUO, 801 DeMun Ave., St. Louis, Mo.

SAN FRANCISCO—Chairman, W. G. Wagener; Secretary, R. V. Howard, Associated Broadcasters, Inc., Mark Hopkins Hotel, San Francisco 6, Calif.

SEATTLE—Chairman, F. B. Mossman; Secretary, E. H. Smith, Apt. K, 1620—14 Ave., Seattle, 22 Wash.

TORONTO—Chairman, E. O. Swan; Secretary, Alexander Bow, Copper Wire Products, Ltd., 137 Roncesvalles Ave., Toronto, Ont., Canada.

TWIN CITIES—Chairman, E. S. Heiser; Secretary, B. R. Hilker, KSTP, St. Paul Hotel, St. Paul, Minn.

WASHINGTON—Chairman, J. D. Wallace; Secretary, F. W. Albertson, c/o Dow and Lohnes, Munsey Bldg., Washington 4, D. C.

WILLIAMSPORT—Chairman, H. E. Smithgall, Jr.; Secretary, K. E. Carl, Williamsport Technical Institute, Williamsport, Pa.

Institute News and Radio Notes

Constitutional Amendment Section

In the September issue of the PROCEEDINGS, there appeared a letter by Mr. H. P. Westman dealing with the subject of dues, in which letter he advocated a "uniform level" system for all members, regardless of grade, excepting students. Mr. Westman, after writing this letter, implemented his ideas by drawing up in petition form a proposed amendment to the Institute Constitution. He has secured the required number of petition signers.

This petition arrived in the Institute office on August 15, too late for its inclusion in the September issue, and also too late for handling with other recently submitted amendments in a fashion suggested by Mr. Westman without delaying seriously the submission of a previously offered constitutional amendment or confusing, by conflicting proposals, the issue of dues in the minds of the prospective voters.

The matter was submitted to legal counsel and possible methods of handling two sequentially received amendments dealing with one subject were discussed. Legal counsel advises that the dues amendments proposed by the Board and published with announcement of impending submission should be put to a vote according to the schedule as drawn up and made public: and that after the returns from this ballot have been received and counted, the amendments proposed by Mr. Westman should be put to a vote. Although this procedure may not be entirely to the liking of the signers of the latter petition, Counsel advises that it is the only legal method with the avoidance of measures that would be to the disadvantage of both amendments.

Members of the Executive Committee and the Chairman of the Constitution and Laws Committee have consulted with Mr. Westman and have tried earnestly and diligently to find a method of handling the matter that would be satisfactory both to Mr. Westman and to the Board, but were unable to do so. The Board, at its meeting on September 6, accepted the advice of Counsel Zeamans, who was there present, and approved the plan of successive presentation for vote of the dues plans in the order in which they were prepared and received. The membership may therefore expect at the earliest possible date (probably late in December) to receive a ballot enabling them to vote upon the plan submitted in the petition submitted by Mr. Westman.

The letter received by the Board, and the petition are as follows:

CONSTITUTIONAL-AMENDMENT PETITION

To the Board of Directors

"The undersigned voting member(s) of the Institute of Radio Engineers hereby petition(s) for the following amendment of the Constitution of the Institute:

"Article IV

"(Replace present Section 1 with the following three Sections.)

"Sec. 1—The entrance fee for all grades shall be $3.00 except that there shall be no entrance fee for Student grade.

"Sec. 2—There shall be no transfer fees.

"Sec. 3—The annual dues for all grades shall be $10.00 except for Student which shall be $3.00 and except for Associate which shall be $7.00 for each year that is within the first five years of membership in any grade or grades other than Student. Thereafter, the annual Associate dues shall be $10.00 beginning the January first following. The clause increasing Associate dues from $7.00 to $10.00 shall take effect on January 1, 1946."

(The 14 indicated pages of this petition were received on August 15, 1944.)

Page 1

Frank E. Canavaciol — Forest Hills, N. Y.

Page 2

P. L. Hoover — Cleveland, Ohio

Page 3

Palmer H. Craig — Gainesville, Fla.

Page 4

Robert P. Siskind — West Lafayette, Ind.

Page 5

John C. Stroebel — Clearfield, Pa.
Harold P. Westman — Hempstead Gardens, L. I., N. Y.

Page 6

Cornelius G. Brennecke — Bethlehem, Pa.

Page 7

Lawrence R. Quarles — Proffit, Va.

Page 8

Frederick W. Grover — Schenectady, N. Y.

Page 9

Hiram D. Harris — Troy, N. Y.
Warren C. Stoker — Troy, N. Y.

Page 10

R. C. Gorham — Pittsburgh, Pa.
L. E. Williams — Bridgeville, Pa.

Page 11

Walter R. Jones — Emporium, Pa.
Marcus A. Acheson — Emporium, Pa.
G. T. Gunnell — Emporium, Pa.
B. S. Ellefson — Bayside, L. I., N. Y.
O. H. Biggs — Beverly, Mass.
Virgil M. Graham — Williamsport, Pa.

Page 12

H. O. Peterson — Riverhead, L. I., N. Y.
K. G. MacLean — Riverhead, L. I., N. Y.
R. W. George — Riverhead, L. I., N. Y.
J. B. Atwood — Riverhead, L. I., N. Y.
DeWitt R. Goddard — Riverhead, L. I., N. Y.
G. S. Wickizer — Riverhead, L. I., N. Y.
Robert E. Schock — Riverhead, L. I., N. Y.
Arthur M. Braaten — Riverhead, L. I., N. Y.

Ferdinand J. W. Schoenborn	Riverhead, L. I., N. Y.
Bertram Trevor	Riverhead, L. I., N. Y.

Page 13

George T. Royden	Newark, N. J.
Francis H. Tratt	Verona, N. J.
John S. McKechnie	West Caldwell, N. J.
Hugo Romander	Newark, N. J.
William W. Macalpine	East Orange, N. J.
Herman P. Miller, Jr.	East Orange, N. J.
Robert A. Hampshire	West Caldwell, N. J.
L. J. Biskner	Newark, N. J.
S. D. Browning	Newark, N. J.

Page 14

Edward F. DeMers	Rochester, N. Y.
Harold Goldberg	Rochester, N. Y.
James A. Krumhansel	Ithaca, N. Y.
Ken L. Henderson	Rochester, N. Y.
Newton H. Odell	Rochester, N. Y.
Roy S. Anderson	Rochester, N. Y.
E. S. Wilson	Rochester, N. Y.
Arthur E. Newlon	Rochester, N. Y.
George R. Town	Rochester, N. Y.
R. H. Freck	Rochester, N. Y.
Howard H. Brauer	Rochester, N. Y.
Edward D. Chalmers	Rochester, N. Y.
George H. Haupt	Rochester, N. Y.

NOTE: Since this petition was originally received, additional copies, containing the names of 57 more signers have been received, too late for inclusion in this issue.

Executive Committee

August 2 Meeting: At the regular meeting of the Executive Committee, which was held on August 2, 1944, the following were present: H. M. Turner, president; Alfred N. Goldsmith, editor; R. A. Heising, treasurer; Haraden Pratt, secretary; H. A. Wheeler, and W. B. Cowilich, assistant secretary.

Membership: The following transfers and applications for membership were unanimously approved: for transfer to Senior Member grade, J. L. Barnes, P. H. Craig, R. W. George, A. G. Kandoian, B. F. Miller, J. F. Palmquist, and Paul Ware; for admission to Senior Member grade, B. V. Bowden, L. P. Graner, F. L. Holloway, O. H. Schmitt, and F. J. Singer; for transfer to Member grade, Wilfred Arcand, F. P. Barnes, F. R. Brace, W. N. Brown, Jr., H. A. Burroughs, W. R. Hill, Jr., Maurice Jacobs, H. D. Johnson, A. A. Kees, P. W. Klipsch, R. S. Marston, George Rappaport, L. P. Richmond, and Simeon Weston; for admission to Member grade, Harland Bass, Harold Bechtol, Sidney Berg, J. S. Cohen, D. F. Folland, C. A. Hansen, Alexander Horvath, R. J. Iverson, J. S. Kiss, R. H. Langstaff B. M. Meador, H. M. O'Bryan, L. J. Peterson, P. M. Reyling, A. D. Sobel, W. A. Tyrrell, and R. H. Van Horn; Associate grade, 125; and Student grade, 19.

Sections

Chicago and Montreal: President Turner stated that letters, reviewing the year's activities of the Chicago and Montreal Sections, had been submitted to him. These letters were referred to Mr. H. A. Wheeler, with the request that they be given further consideration and that a report on them be given at a later date.

Ottawa: A petition, in good order, requesting the formation of an Ottawa Section was considered.

Following the discussion, the establishment of the Ottawa Section was unanimously recommended to the Board with the provision that the boundaries of the area of that Section be decided at a later date.

Mr. H. A. Wheeler and the assistant secretary were requested to communicate with the chairmen of the Ottawa, Montreal, and Toronto Sections in arranging the territory of the new Section, and to consider the preference of members and the inclusion of entire electoral districts as guiding factors.

R. A. Hackbush: A petition, in good order, nominating Vice-President R. A. Hackbusch for the office of President for 1945 was reported. As a result of the petition, the name of Vice-President Hackbusch, as a candidate for President, was added on the ballot.

Ballots: As information of value to the membership, it was decided to designate on the ballot, if such step is legal in the opinion of the Institute's General Counsel, those candidates who are Board nominees and those nominated by independent petition. (General-Counsel Zeamans stated that not only are such designations on the ballot legal, but also the number of members signing the petitions may properly be given.)

Petitions: It was decided that, as a policy, all petitions including the names of signers and their addresses (city and state only) would be published in the PROCEEDINGS.

Publicity on Voting: The recommendation was made to the Board of Directors that, as a general policy, a detailed analysis of the voting on constitutional amendments should be published in the PROCEEDINGS.

Institute Representative in College: Unanimous approval was given to the recommendation to the Board that Professor Harold Wolf be appointed to succeed the late Professor Maxwell Henry, as Institute Representative at the City College of New York.

American Standards Association: President Turner called attention to the ASA July 24, 1944, letter inviting him to attend a meeting on August 16, 1944, called for the purpose of developing an American War Standards on Methods of Measuring Radio Noise, and stated that he planned to be present at that meeting.

Board of Directors Nomination Petition

The editorial department of the Institute received the following petition, signed by 42 members in good standing, too late to be included in this issue, with the names and addresses of all signers:

To the Board of Directors

"We nominate Lewis M. Clement, Fellow member of the I.R.E., as a candidate for the Board of Directors and request that his name be placed on the ballots to be voted this year.

1944 Rochester Fall Meeting

The Sheraton Hotel (formerly Sagamore)
Rochester, New York

November 13 and 14, 1944

Program

Monday, November 13

8:30 A.M.

REGISTRATION

9:30 A.M.

TECHNICAL SESSION

"The Reactance Theorem for a Resonator," by W. R. MacLean, Polytechnic Institute of Brooklyn.

"A Resonant-Cavity Method for Measuring Dielectric Properties at Ultra-High Frequencies," by C. N. Works, T. W. Dakin, and F. G. Boggs, Westinghouse Electric and Manufacturing Company.

"The RCA Laboratories at Princeton," by E. W. Engstrom, Radio Corporation of America.

12:30 P.M.

GROUP LUNCHEON

2:00 P.M.

TECHNICAL SESSION

"Low-Frequency Compensation of Multistage Video Amplifiers," by M. J. Larson and A. E. Newlon, Stromberg-Carlson Company.

"Trends in Receiving-Tube Design and Application," by L. R. Martin, Radio Corporation of America.

"Standardization of Capacitors for Civilian Equipment," by J. I. Cornell, Solar Manufacturing Corporation.

4:00 P.M.

COMMITTEE MEETINGS

6:30 P.M.

GROUP DINNER

8:15 P.M.

TECHNICAL SESSION

"One Look Backwards—and Two Ahead," by K. W. Jarvis, Sheridan Electro Corporation.

Tuesday, November 14

8:30 A.M.

REGISTRATION

9:00 A.M.

TECHNICAL SESSION

"Report of RMA Director of Engineering," by W. R. G. Baker.

"The Organization of Research in the Radio Industry after the War," by Rupert Maclaurin, Massachusetts Institute of Technology.

"Electronic-Tube Trends," by R. M. Wise, Sylvania Electric Products Inc.

12:30 P.M.

GROUP LUNCHEON

2:00 P.M.

TECHNICAL SESSION

"Silicone Products of Interest to the Radio Industry," by Shailer L. Bass and T. A. Kauppi, Dow Corning Corporation.

"Designing Thoriated Tungsten Cathodes," by H. J. Dailey, Westinghouse Electric and Manufacturing Company.

4:00 P.M.

COMMITTEE MEETINGS

6:30 P.M.

STAG BANQUET

F. S. Barton—Toastmaster.

Major-General Roger B. Colton—Speaker

Correspondence

Phase-Shift Oscillators*

The above paper has been very interesting and useful, and the one-tube resistance-capacitance oscillators are quite stable and satisfactory. Certainly the authors are to be congratulated upon the development of this new type of oscillator.

In the course of our work, further calculations were made and compared with the results given in this paper. In this regard I should like to call your attention to the amplification of the fourth and fifth lines of Fig. 3.

(1) *Fourth line*—(four-mesh network, capacitors in series, resistors in parallel in the special case where only two condensers are ganged). The attenuation formula given for this case is

$$\frac{9(C/C_1)^3+33(C/C_1)^2+55(C/C_1)+90+84(C_1/C)}{(4+3(C/C_1)^2}.$$

From my calculations, the attenuation is

$$\frac{9(C/C_1)^3+114(C/C_1)^2+352(C/C_1)+342+84(C_1/C)}{(4+3(C/C_1)^2}.$$

The amplification curve of Fig. 5 agrees with your formula, not with mine, which shows that, if there is a mistake, it is not merely a typographical error. If we choose three particular values for C/C_1, say 0.1, 1., and 10, the attenuations by your formula are respectively 50.6, 5.53, and 11.2, whereas by my formula they are respectively 65.8, 18.4, and 21. It is almost unnecessary to make any calculation to convince one's self that although the frequency is not the same for a network comprising four equal condensers in series and four equal resistances in parallel and for a network where the condensers are in parallel and the resistances in series, the attenuation is the same in both cases. As an illustration of this, the attenuation given by my formula when $C/C_1=1$ (namely, 18.4) is equal to the attenuation given in the sixth line of your chart for a four-mesh network with resistances in series and condensers in parallel.

(2) *Fifth line*—(three-mesh network, resistors in series, capacitors in parallel). You give the attenuation, in this case, as being 5, whereas in the first line, when the capacitors are in series and the resistances in parallel, you give an attenuation of 29. It seems evident that the attenuation is the same in both cases.

In the network of the first line the frequency being $1/(2\pi\sqrt{6}\,RC)$ the impedance of the condensers is $\sqrt{6}\,R$; that is, in each mesh the ratio

$$\frac{\text{impedance of the leg in series}}{\text{impedance of the leg in parallel}}=\sqrt{6}.$$

In the network of the fifth line, the frequency being the impedance of the condensers is $R/\sqrt{6}$. In each mesh the ratio

$$\frac{\text{impedance of the leg in series}}{\text{impedance of the leg in parallel}}=\sqrt{6}.$$

This ratio determines completely the attenuation and is the same in both cases.

As the paper was published several years ago, I feel sure that if there really is a mistake, attention has already been called to it. In any case, as this paper has already proved very useful to us, I feel that if a correction is necessary, it should be published so as to complete the usefulness of your discovery.

A. BLANCHARD
Schlumberger Well Surveying Corporation
Houston, Texas

(NOTE: This letter is published with the concurrence of the authors of the paper on which it comments. *The Editor*)

* E. L. Ginzton and L. M. Hollingsworth, "Phase-shift oscillators," PROC. I.R.E., vol. 29, pp. 49–49; February, 1941.

Grounded-Grid Amplifiers

Two articles on grounded-grid amplifiers, appearing in recent issues of this magazine, gave some calculations on the ratio of signal to noise for the grounded-grid circuit. Some time ago I had occasion to carry out similar calculations and wish to point out in this letter that a careful analysis shows the grounded-grid amplifier to have exactly the same signal-to-noise ratio as the ordinary grid-fed amplifier. The analysis also shows that the noise power originating in succeeding stages is of equal importance in both types of amplifier; it only depends on the grid-to-grid gain per stage.

For example, if the noise of all succeeding stages is expressed in terms of a constant-current generator in parallel with the plate-load resistor of the first stage, as shown in Fig. 1, then, one way of expressing the effect of this added noise is to increase the equivalent noise resistance of the first tube as follows:

$$req=req_1+\frac{1}{R_{N}g_m{}^2}.$$

Both articles that have been published assumed *ad hoc* that the noise-equivalent resistance for a tube used as a grounded-grid amplifier was the same as when used in a conventional circuit. Since the grounded-grid amplifier is a feedback circuit, this is by no means evident; however, it can be proved to be true.

It has been shown[1] that the shot noise originating within a tube may be represented by a constant-current generator acting between the cathode and the plate of the tube. $\bar{i}^2=\theta/\sigma\cdot 4kT_k g_m\Delta f$ where $\theta\approx 2/3$, and for conventional tubes σ lies between 0.5 to 1. g_m is the transconductance of the tube and T_k is temperature of the cathode.

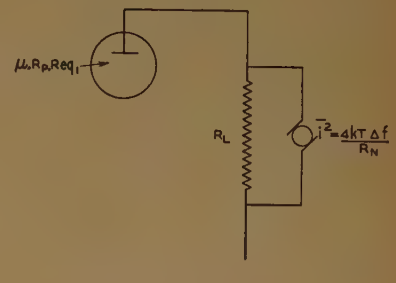


Fig. 1.

The shot noise may also be represented by a constant-voltage generator placed in the grid circuit of the tube. It is convenient to express this voltage in terms of an equivalent resistor $(R_{eq}+j0)$ required to generate the same mean-square voltage fluctuations. The value of this resistor will be shown not to depend on feedback.

When no feedback is present, we have by definition

$$\begin{aligned}\bar{e}_{eq}{}^2&=\bar{i}^2/g_m{}^2\\&=(\theta/\sigma\cdot T_k/T\cdot 1/g_m)4kT\Delta f\\&=4kTR_{eq}\Delta f.\end{aligned}$$

With this notation, $\bar{i}^2=4KTR_{eq}g_m{}^2\Delta f$. The above treatment of noise currents is justified since we can assume that the formulas apply to the currents in each infinitesimal frequency interval and since these currents are uncorrelated (incoherent) the resultant current squared in any large frequency interval Δf is obtained, by adding their squares.

When feedback is present, we must consider the circuit shown in Fig. 2.

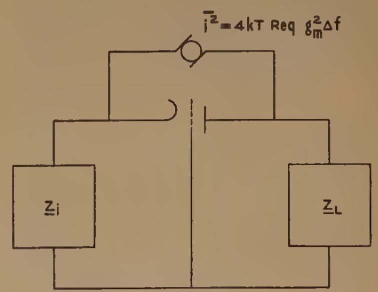


Fig. 2.

The equivalent circuit may be drawn as shown in Fig. 3.

[1] D. O. North, "Fluctuations in space-charge-limited currents at moderately high frequencies, Part II, Diodes and negative-grid triodes," *RCA Rev.*, vol. 4, pp. 441–473; April, 1941; equation (41a), p. 470.

The general method of calculating the noise currents has already been indicated above. Specifically, the method consists in taking the noise current in a small frequency interval and allowing this interval to become infinitesimally narrow, say between f and $f+\delta f$ the noise current in it approaches that of a pure sine wave of frequency f. The noise power remains proportional to the width of the interval δf. Regular circuit analysis may now be applied to these currents.

To obtain the total noise power, we finally integrate the squares of the magnitudes of these noise currents over all frequencies.[2]

Thus, at the frequency f the mesh cur-

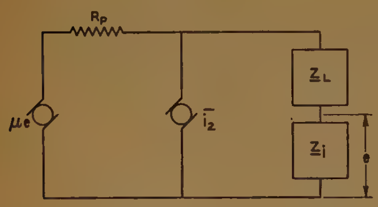


Fig. 3.

rents through the load impedance Z_L in the above equivalent circuit are

$$\underline{i_L} = \sqrt{\overline{i_f^2}}\,\frac{R_p}{R_p + \underline{Z}_L + (1+\mu)\underline{Z}_i}.$$

Integrating the squares of the magnitudes of these currents over all frequencies we obtain

$$|\underline{i_L}|^2 = 4kTR_{\text{eq}}g_m{}^2\Delta f\,\frac{R_p{}^2}{|R_p+\underline{Z}_L+(1+\mu)\underline{Z}_i|^2}.$$

where we have assumed that the bandwidth of the coupling circuits, represented by the frequency-sensitive component is much broader than the over-all bandwidth of the receiver f.

It can easily be shown that a constant-voltage generator at the grid of the tube whose voltage is given by $\bar{e}_{\text{eq}}{}^2 = 4kTR_{\text{eq}}\Delta f$ gives rise to the same current $|\underline{i_L}|^2$ through the load impedance.

Carrying through similar analyses for other sources of noise and comparing the resulting total noise power at the output of the amplifier to that due to a given signal shows this ratio to be the same as for ordinary grid-fed amplifiers. Thus all conclusions drawn by Herold[3] in his excellent paper also apply to the grounded-grid amplifier.

Leon Katz
Westinghouse Electric and
Manufacturing Company
East Pittsburgh, Pennsylvania

[2] This is a method commonly used in optics when a nonmonochromatic beam of light is subject to interference. The method has also been recently applied to circuit analysis of noise by M. J. O. Strutt and A. van Der Ziel "Methods for compensating the effects of various kinds of shot noise in electron tubes and attached circuits," Physica, vol. 8, pp. 1–22; 1941.

[3] E. W. Herold, "An analysis of the signal-to-noise ratio of ultra-high frequency receivers," *RCA Rev.*, vol. 6, pp. 302–329; January, 1942.

Equivalent-Plate-Circuit Theorem*

I have read with considerable interest Mr. Stockman's letter, particularly the remark about an error concerning plate dissipation that appeared in some books recently published, etc. My own book, "Graphical Constructions for Vacuum Tube Circuits," contains this error.

As Mr. Stockman points out, the equivalent plate circuit for a vacuum tube can be employed to give the plate dissipation in the tube. Upon rereading the offending passage in my book, I find that I simply state it cannot be employed. The wording was unfortunate; what I meant to indicate was that the equivalent plate circuit, *by itself*, cannot yield this information. The circuit generally merely shows a generator whose internal resistance is r_p, and whose generated voltage is μE_g, in series with an external load impedance Z_L. The current that flows is alternating current and of value $I_p = \mu E_g/(r_p + Z_L)$.

The internal loss and hence plate dissipation should, therefore, be simply $(I_p)^2 r_p$. This, however, would be an incorrect conclusion, and is the point I intended to convey. This fact has been stressed by Mr. Stockman's associate, Professor Chaffee on page 220 of his book, "Theory of Thermionic Vacuum Tubes."

As Mr. Stockman points out, the plate dissipation is equal to the direct-current power input $e_b i_b$, minus the portion converted into a hypothetical alternating-current power $\mu E_g I_p$, plus the portion of this power converted into heating within the tube; namely, $(I_p)^2 r_p$. Thus the latter term is but *one* of *three* in the bookkeeping process required to evaluate the plate dissipation, rather than the only term, as the equivalent plate circuit might by itself suggest. The equation given by Mr. Stockman is $P_p - P_g + r_p I_p{}^2 =$ plate dissipation. This can be very simply rewritten as $P_p - (P_g - r_p I_p{}^2) =$ plate dissipation. The term $(P_g - r_p I_p{}^2)$ is obviously the power expended in the load impedance Z_L, so that the plate dissipation can be more simply expressed as the direct-current power input P_p minus the alternating-current-power output in Z_L.

This is the form normally employed unless one desires to show that the equivalent plate circuit can be used. Not only is it simpler, but it holds in all cases, even for such nonlinear operation that the correct equivalent plate circuit becomes far too complicated to employ, such as in class C amplifiers. Moreover, it can be determined experimentally and with little difficulty as a general rule. On the other hand, Mr. Stockman's method has the merit of breaking up the power relations into several terms, and thus shedding light on the internal action of the tube.

On another point that Mr. Stockman stresses, namely the use of the expression "equivalent-plate-circuit equation" instead of the "equivalent-plate-circuit theorem," I find myself in disagreement.

In the last analysis, whether one calls a property a theorem or an equation is a matter of opinion. To the perspicacious, it may be but an equation; to the ordinary mind it may appear to have the force of a theorem. Often, upon more mature consideration, that which at first appeared obvious begins to appear less and less self-evident. It is interesting to note that Professor Chaffee, in the book previously cited, considered the equivalent plate circuit a theorem.

Many theorems have been developed in network theory. To cite but four, we have the superposition theorem, the reciprocity theorem, the compensation theorem, and Thevenin's theorem. The first two are obvious results of the properties of determinants, yet we choose to call them theorems because of the important physical implications that they convey, even though they apply to linear networks only.

The last two theorems seem in some ways even more important to me because they appear to apply to nonlinear circuits as well. The compensation theorem states that as far as the rest of the circuit is concerned, any impedance voltage drop can be replaced by a suitable voltage source, which will compensate for the removal of the impedance it replaces.

This, in a way, appears obvious, yet its application to switching problems and variable circuit parameters has enabled these problems to be solved by the methods of ordinary circuit analysis. The equivalent circuit is such an important corollary of the compensation theorem that it warrants being considered a theorem, in my estimation.

It indicates that in general a time variable resistance in series with a fixed resistance and a fixed voltage, may be replaced by a fixed resistance and variable voltage in series with the fixed voltage and original

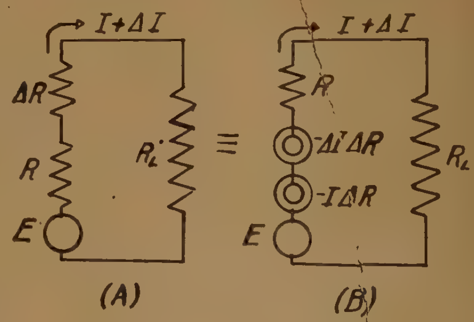


Fig. 1.

fixed resistance. Thus Fig. 1(A) which has a variable resistance ΔR is equivalent to Fig. 1(B). Normally the generator $(-\Delta I\Delta R)$ is disregarded. However, if ΔR is a sinusoidally time-variable resistance, then $(-\Delta I\Delta R)$ is not negligible, particularly as regards dissipation. It will be found that it can be resolved into a whole host of generators: one generating additional direct current, and the others generating harmonics of the fundamental voltage $(-I\Delta R)$.

The interesting thing is that the cooling effect owing to the presence of the additional direct-current generator is as great as that owing to the presence of the alternating-current generator $(-I\Delta R)$. Mr. Stockman's

* Harry Stockman, "The validity of the equivalent plate-circuit theorem for power calculations," Proc. I.R.E., vol. 32, p. 373; June, 1944.

method can be employed to find the internal dissipation in $(R+\Delta R)$, although the method of subtracting the *total* output power in R_L from the total input power gives the dissipation with more dispatch.

One point that should be stressed is that an equivalent circuit can be found (at least theoretically) even for the case where R is a function of the current (nonlinear), as well as time variable in the portion ΔR. Thus the vacuum tube, even over an extended range of operation, where it is not only a time variable but also a nonlinear resistance, can at least theoretically be represented by a linear circuit.

The case of the linear constant-mu tube, discussed by Mr. Stockman, is of interest. Suppose that we wish to represent as it a variable resistance, similar to a carbon button microphone. For ΔI to be a pure sinusoid of frequency $\omega/2\pi$, ΔR must have the form

$$\Delta R = \frac{-(R+R_L)a \sin \omega t}{b + a \sin \omega t}$$

where R is the quiescent resistance of the device $(\Delta R=0)$, R_L is the load resistance, and a and b are constants. In the case of the linear vacuum tube, R is the so-called direct-current resistance of the tube, i.e., the reciprocal slope of the secant line joining the origin to the quiescent point. Also, $b=E_{bb}+\mu E_c=E$, where E_{bb} is the total applied direct voltage and E_c is the bias, while $a=\mu E_g$, where E_g is the peak value of the sinusoidal voltage applied to the grid of the tube.

The equivalent circuit is as shown in Fig. 2. The current I_b is the direct-current component determined by the quiescent point (values of E_{bb} and E_c), and $I_{ac}(=\Delta I)$ is determined by the grid excitation voltage *and* E_{bb} *and* E, *as shown.* This circuit can be used to calculate the plate dissipation, if it be noted that the alternating-current

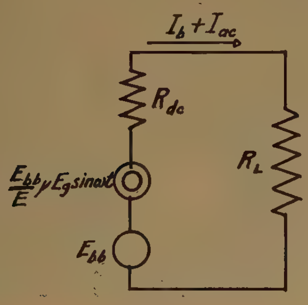


Fig. 2.

generator derives its power from E_{bb}, and therefore produces a cooling effect.

Unfortunately, both the alternating voltage E_{bb}/E, $\mu E_g \sin \omega t$, and R_{dc} are functions of E_{bb} and E_c, and for every choice of quiescent point a different value for the above two quantities would have to be used. In addition, it will be found that R_{dc} is a function of R_L, as well. It is here that the more usual equivalent plate circuit shows its value. Owing to lack of space, I have not indicated the steps that lead to the equivalent circuit of Fig. 2, but among them is an expression $R_{dc}+R_L=(E_{bb}(r_p+R_L)/E)$ where r_p is the variational or alternating-current resistance of the tube, and is independent of the position of the quiescent point (for a linear tube). If this be substituted in the previous equation, one obtains $I_{ac}=\mu e_g/(r_p+R_L)$ which is the well-known equivalent-plate-circuit theorem and indicates that for a linear tube I_{ac} is independent of the quiescent point.

This is an important simplification. While the tube is a kind of variable resistance, it differs from the carbon button microphone, for example, in that the control agency, the voltage applied to the grid, can be regarded as a voltage, μ times as great, acting in the plate circuit. This action produces a direct-current resistance that is a function of the position of the quiescent point and of the value of R_L, which is a complication, but at the same time it allows another resistance, the r_p of the tube, to be employed, and r_p is independent of the above factors.

The simplified equivalent circuit does not, *of itself*, give the plate dissipation, whereas the circuit shown in Fig. 2 does. The joker in the latter case, however, is that R_{dc} is not a fixed value, but depends upon E_{bb}, E_c, and R_L, and must be calculated first before the dissipation can be found.

It is not necessary, of course, to derive the simplified equivalent plate circuit by the method I chose. The ordinary power-series method is in general more simple and hence more desirable. What I wanted to show is that if the tube is regarded as a variable resistance, and treated like the carbon button microphone, then an equivalent linear circuit can be found, which is a step forward. What I further wanted to show was that an even simpler equivalent circuit can be found. This is another step forward, and to my mind, further justifies the expression "equivalent-plate-circuit theorem."

Albert Preisman
Capitol Radio Engineering Institute
Washington, D. C.

I conclude that, except for a question of terminology, Mr. Preisman and I are in agreement about the technical points given in my original letter. This letter, by the way, described the new method as a teaching aid and did not suggest it as a substitute to conventional computation methods.

The question of terminology refers to my statement that it is *doubtful* whether the term "equivalent-plate-circuit-theorem" is justified for (4), in my letter referred to as the "equivalent-plate-circuit equation." Mr. Preisman objects to this, and I think he has a good point there, as the term he defends has always been accepted as the correct one. But as Mr. Preisman says with reference to this case, the choice between "theorem" and "equation" is a matter of opinion.

Mr. Preisman gives an interesting discussion of the compensation theorem, by means of which he obtains the simple equivalent plate circuit. This procedure points towards "theorem" as being more appropriate than "equation." As an example to the contrary, consider the simple classroom routine of presenting a tube with defined coefficients which are constant. Thus the grid contributes g_mE_g and the plate g_pE_p, so that the plate current becomes $g_mE_g+g_pE_p$. In this case, I should prefer the term "equation," as there is no direct need for bringing up the theorem aspect.

Personally, I like "equivalent-plate-circuit-equation" because it does not "scare the daylights" out of the bottom-group students to the same extent as "equivalent-plate-circuit-theorem." I wish an I.R.E. or A.I.E.E. committee could be formed to settle the question of proper names for important laws, theorems, transformations, and equations in the electronic field. No doubt such an action would be welcomed by many workers in the field.

I thank Mr. Preisman for having brought up a number of interesting and important points. In the introduction to his letter, he mentions his book about graphical construction, and I take this opportunity to congratulate him for this excellent contribution to the literature in the field of radio engineering.

Harry Stockman
Cruft Laboratory, Harvard University
Cambridge, Mass.

I.R.E. People

Allen B. Du Mont

During its 120th commencement exercises, Rensselaer Polytechnic Institute conferred the honorary degree of Doctor of Engineering upon Allen B. Du Mont, president of the Allen B. Du Mont Laboratories, Inc., of Passaic, New Jersey, and graduate of the class of 1924.

Allen B. Du Mont

Mr. Du Mont was cited particularly as "a pioneer in the development and use of the cathode-ray tube, which today is the heart of the weapon, radar. Through his efforts," the citation read, "the cathode-ray tube was ready for production on a quantity basis when it was needed in the war.

"He has also improved the science and art of television and promoted it in public consciousness through the ownership in New York City of one of the first television stations," the citation continued. "Scores of patents reflect his career as a scientist, engineer, inventor, and industrialist."

Addressing the graduating class, Mr. Du

Mont declared: "For the engineer and scientist, tomorrow's world means change and progress on a global and positively fantastic scale. Because of the war, technological advances already amount to twenty-five years of usual peacetime progress. In many fields, the textbooks you have been studying are probably already two to five years behind times." Mr. Du Mont urged the graduates to think in creative terms, to be sales- and business-minded, and to take a greater part in world leadership, so as to reap the greater reward to which their training and genius entitles them.

Mr. Du Mont joined the Institute of Radio Engineers as a Member in 1930 and transferred to Fellow grade in 1931.

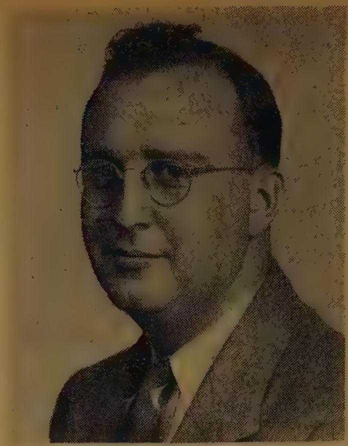
RCA Personnel Changes

Edmund A. Laport (A'25-M'27), active in installations of broadcast transmitters both here and abroad, has been appointed staff engineer for international communications systems and special apparatus at Camden, New Jersey, it was announced recently by Dr. C. B. Jolliffe (M'28-F'30), chief engineer of the RCA Victor Division, Radio Corporation of America. At the same time announcement was made of the appointment of James B. Knox (A'34) to succeed Mr. Laport as chief engineer for engineering products at RCA's Canadian subsidiary, RCA Victor, Ltd.

In his new position, Mr. Laport will be responsible for the company's engineering in connection with international communications systems and engineering products for sale in the international field.

Edmund A. Laport

Born in Nashua, N. H., Mr. Laport was an amateur radio operator in his teens and a commercial radio operator at KDKF, New York, in 1921. From 1924 to 1932 he was active for RCA in the foreign field, installing important transmitter equipment. In 1928 he built three mobile railway transmitter stations for the Chinese government at Peking and Tsientsin. Later he installed two of Italy's foremost standard broadcast stations; Rome's 1 RO (located near the Anzio beachhead) and Milan's 1 MI.



James B. Knox

The American broadcast industry is acquainted with his work from his installations of 50-kilowatt broadcast transmitters for WJZ, WSN, WBZ, WOAI (San Antonio), and CBK (Canadian Broadcasting System's Watrous, Saskatchewan, transmitter). Among other installations made by Mr. Laport are Asheville's WWNC, Philadelphia's WRAX, Reading's WEEO, Chicago's WCFL, and Hollywood's KMTR. He also is the author of many technical papers and the holder of ten patents for antenna and transmission apparatus.

In 1938 he became chief engineer for the Canadian subsidiary's department of design and development of transmitters, building up an engineering staff that has made wartime contributions in the communications field for Canada and her sister nations of the British empire.

In succeeding Mr. Laport in the Canadian post, Mr. Knox brings to his new position an active engineering record both here and abroad. Prior to his appointment he was senior engineer in the engineering products department, under Mr. Laport. Previously, he had been connected with China Airways, Shanghai, and the British General Electric Company and Standard Telephone and Cables Company, both of London. In 1937 he returned to Canada to serve as an official of the radio division, Department of Transport, in the Canadian government until 1941, when he joined RCA Victor.

Philip F. Siling

Philip F. Siling (M'40), assistant chief engineer in charge of broadcasting, Federal Communications Commission, Washington, D. C., has been appointed engineer-in-charge of the frequency bureau of the Radio Corporation of America effective October 1, it was announced by O. S. Schairer, vice-president in charge of RCA Laboratories.

In his new post, Mr. Siling, who has been associated with the FCC for nine years, will handle matters pertaining to frequency allocations and licenses for RCA, its subsidiaries, and services. These activities cover the fields of sound broadcasting, television, international point-to-point communications, marine communications, and experimental operations.

Mr. Siling will maintain offices in the RCA Building, 30 Rockefeller Plaza, New York, and at 1625 K Street, N. W., Washington, D. C. The duties of the engineer-in-charge of the RCA frequency bureau have been administered by Dr. B. E. Shackelford (A'23-M'26-F'38) since the post was relinquished two years ago by Dr. C. B. Jolliffe (M'28-F'30), former chief engineer of the FCC, to become chief engineer of the RCA Victor Division, Camden, New Jersey. Dr. Shackelford will retain general direction of the Bureau's activities. C. E. Pfautz (A'20), is manager of the New York office of the bureau.

Mr. Siling was graduated from Yale University with a bachelor of philosphy degree in electrical engineering. He worked as an engineer with the American Telephone and Telegraph Company from 1917 to 1929, when he became outside plant engineer of the International Telephone and Telegraph Corporation. Shortly thereafter, he was appointed acting plant operations engineer of the same company. In 1931, he became superintendent of materials and supplies for the International Telephone and Telegraph Corporation of South America, with headquarters in Buenos Aires. In 1933, he was named assistant deputy administrator of the National Recovery Administration in charge of codes of the electrical manufacturing industry.

Transferring to the FCC in 1935, Mr. Siling served successively as senior telephone engineer, assistant chief of the international division, and assistant chief engineer in charge of the broadcast division. In addition to his FCC assignments, he has served as assistant secretary of the Interdepartment Radio Advisory Committee, 1937 to 1941, and as secretary, 1941 to date. He also was chairman of the Technical Subcommittee of the Interdepartment Radio Advisory Committee.

Philip F. Siling

Forest S. Mabry

For outstanding work in designing and manufacturing radar equipment, Forrest S. Mabry, section engineer in the radio division of the Westinghouse Electric and Manufacturing Company, Baltimore, Maryland,

was awarded the Order of Merit, the Company's highest honor, by the Board of Directors.

Mr. Mabry, cited "for his resourcefulness in bringing to satisfactory solution many unusual design problems, especially in the field of airborne and shipborne radar; and for his work and the work of those responsible to him for efficient group performance," is a native of Laurel Fork, Virginia. He was graduated from the Westinghouse Technical Night School in 1925, completing a four-year course in electrical engineering in three years, while working as a transformer winder in the East Pittsburgh Works. In 1925 he was assigned to the carrier current-equipment section of the radio division, making installations of this type of equipment throughout eastern United States. When the radio division was moved from Chicopee Falls, Massachusetts, to Baltimore in 1938, Mr. Mabry was made a section engineer here, supervising design of power-line carrier, airborne, land-based, and ship radar. Mr. Mabry has seven patents on radio equipment. He is an Associate member of the I.R.E.

Institute Committees—1944

Admissions

G. T. Royden, *Chairman*

S. L. Bailey
H. H. Beverage
J. L. Callahan
E. F. Carter
E. D. Cook
Lloyd Espenschied
T. T. Goldsmith, Jr.
A. R. Hodges
Knox McIlwain
F. A. Polkinghorn
H. J. Reich
C. E. Scholz
S. W. Seeley
M. E. Strieby
F. D. Webster
R. M. Wise

Awards

W. L. Everitt, *Chairman*

W. L. Barrow
D. D. Israel
R. K. Potter
Haraden Pratt
B. E. Shackelford
H. A. Wheeler
W. C. White

Board of Editors

A. N. Goldsmith, *Chairman*

R. R. Batcher
R. S. Burnap
P. S. Carter
L. M. Clement
E. M. Deloraine
W. G. Dow
E. W. Engstrom
W. L. Everitt
G. W. Gilman
P. C. Goldmark
F. W. Grover
L. B. Headrick
C. M. Jansky, Jr.
J. D. Kraus
D. G. Little
F. B. Llewellyn
S. S. Mackeown
E. L. Nelson
H. F. Olson
H. O. Peterson
G. W. Pickard
R. A. Powers
Haraden Pratt
C. A. Priest
H. J. Reich
P. C. Sandretto
V. W. Sherman
L. C. Smeby
E. C. Wente
H. A. Wheeler
W. C. White
L. E. Whittemore
G. W. Willard
William Wilson
C. J. Young
V. K. Zworykin

Constitution and Laws

R. A. Heising, *Chairman*

Austin Bailey
S. L. Bailey
W. L. Barrow
V. M. Graham
G. T. Royden
W. M. Smith
F. E. Terman
H. R. Zeamans

Education

W. L. Everitt, *Chairman*

A. B. Bronwell
W. H. Campbell
C. C. Chambers
Fulton Cutting
Paul DeMars
Melville Eastham
C. M. Jansky, Jr.
F. H. Kirkpatrick
J. A. Ouimet
B. R. Teare, Jr.
Ernst Weber

Executive

H. M. Turner, *Chairman*
R. A. Heising, *Vice Chairman*
Haraden Pratt, *Secretary*

E. F. Carter
Alfred N. Goldsmith
F. B. Llewellyn
H. A. Wheeler

Investments

R. A. Heising, *Chairman*

Fulton Cutting
Haraden Pratt
H. M. Turner
H. R. Zeamans

Membership

E. D. Cook, *Chairman*

V. J. Andrew
C. R. Barhydt
C. M. Burrill
W. F. Choat
J. M. Clayton
R. I. Cole
J. M. Comer, Jr.
W. C. Copp
W. H. Doherty
A. V. Eastman
W. G. Eaton
A. J. Ebel
W. N. Eldred
D. G. Fink
A. W. Graf
Samuel Gubin
L. B. Headrick
L. G. Hector
J. F. Johnson
R. L. Kelly
J. G. Kreer, Jr.
M. M. Ming
J. A. Morton
Albert Preisman
J. L. Reinartz
Bernard Salzberg
R. B. Shanck
R. L. Snyder
J. C. Stroebel
Sarkes Tarzian
Bertram Trevor
K. S. Van Dyke
H. M. Wagner
Ernst Weber

(Section Secretaries *ex officio*)

Nominations

Ralph Bown, *Chairman*

I. S. Coggeshall
J. V. L. Hogan
O. B. Hanson
F. B. Llewellyn
L. P. Wheeler
W. C. White

Papers Procurement Committee

Dorman D. Israel, *General Chairman*
William L. Everitt, *Vice Chairman*

GROUP CHAIRMEN

J. E. Brown
W. B. Burgess
E. J. Content
Harry Diamond
E. T. Dickey
J. K. Johnson
C. J. Madsen
D. H. Moore
J. R. Nelson
H. J. Tyzzer
W. C. White

Papers

F. B. Llewellyn, *Chairman*

H. A. Affel
W. L. Barrow
H. A. Chinn
J. K. Clapp
I. S. Coggeshall
M. G. Crosby
F. W. Cunningham
R. B. Dome
E. B. Ferrell
D. G. Fink
H. S. Frazier
Stanford Goldman
F. W. Grover
O. B. Hanson
E. W. Herold
J. V. L. Hogan
F. V. Hunt
Harley Iams
L. F. Jones
J. G. Kreer, Jr.
Emil Labin
F. R. Lack
H. C. Leuteritz
D. K. Martin
Knox McIlwain
H. R. Mimno
I. E. Mouromtseff
G. G. Muller
A. F. Murray
D. O. North
A. F. Pomeroy
J. R. Poppele
Simon Ramo
F. X. Rettenmeyer
S. A. Schelkunoff
D. B. Sinclair
H. M. Turner
Dayton Ulrey
K. S. VanDyke
E. K. Van Tassel
J. R. Whinnery
Irving Wolff
J. W. Wright
H. R. Zeamans

Public Relations

I. S. Coggeshall, *Chairman*

O. H. Caldwell
E. K. Cohan
P. B. Findley
Augustin Frigon
George Grammer
O. B. Hanson
Keith Henney
D. D. Israel
T. R. Kennedy, Jr.
J. R. Poppele
R. C. Poulter

Sections

R. A. Heising, *Chairman*

C. N. Anderson
E. D. Cook
V. M. Graham
J. K. Johnson
J. H. Miller
F. A. Polkinghorn
G. T. Royden
B. E. Shackelford

(Section Chairmen, *ex officio*)

Tellers

H. F. Dart, *Chairman*

E. J. Content
T. T. Goldsmith, Jr.
G. B. Hoadley
C. E. Scholz

Technical Committees

May 1, 1944, to May 1, 1945

Annual Review

L. E. Whittemore, *Chairman*

Andrew Alford, E. A. Guillemin, R. S. Burnap, R. F. Guy, C. R. Burrows, Keith Henney, W. G. Cady, I. J. Kaar, C. C. Chambers, G. G. Muller, I. S. Coggeshall, H. A. Wheeler, L. F. Curtis, C. J. Young

(Plus Member-at-Large)

Circuits Committee

E. A. Guillemin, *Chairman*

H. W. Bode, J. M. Miller, Cledo Brunetti, C. A. Nietzert, F. C. Everett, A. F. Pomeroy, W. L. Everitt, J. B. Russell, Jr., L. A. Kelley, Stuart W. Seeley, Knox McIlwain, W. N. Tuttle, H. A. Wheeler

Electroacoustics

G. G. Muller, *Chairman*

S. J. Begun, Benjamin Olney, F. V. Hunt, H. F. Olson, V. N. James, H. H. Scott, G. M. Nixon, L. J. Sivian

Electronics

R. S. Burnap, *Chairman*

E. L. Chaffee, L. S. Nergaard, K. C. Dewalt, G. D. O'Neill, W. G. Dow, H. W. Parker, R. L. Freeman, H. J. Reich, A. M. Glover, A. C. Rockwood, T. T. Goldsmith, Jr., J. B. Russell, Jr., L. B. Headrick, Bernard Salzberg, E. C. Homer, A. L. Samuel, D. R. Hull, J. R. Steen, S. B. Ingram, C. M. Wheeler, D. E. Marshall, J. R. Wilson, J. A. Morton, Irving Wolff, I. E. Mouromtseff, Jack Yolles

Facsimile

C. J. Young, *Chairman*

Maurice Artzt, J. V. L. Hogan, E. P. Bancroft, H. J. Lavery, J. C. Barnes, B. V. Magee, F. R. Brick, Jr., Pierre Mertz, W. A. R. Brown, L. D. Prehn, J. J. Callahan, G. D. Robinson, J. L. Callahan, W. L. Roe, G. V. Dillenback, L. A. Smith, R. J. Wise

Frequency Modulation

C. C. Chambers, *Chairman*

J. E. Brown, V. D. Landon, W. F. Cotter, H. B. Marvin, M. G. Crosby, D. E. Noble, W. L. Everitt, B. E. Shackelford, R. F. Guy, D. B. Smith, C. M. Jansky, Jr., L. P. Wheeler, W. D. White

Radio Receivers

L. F. Curtis, *Chairman*

G. L. Beers, C. J. Franks, W. M. Breazeale, A. R. Hodges, W. F. Cotter, J. K. Johnson, Harry Diamond, Garrard Mountjoy, W. L. Dunn, H. O. Peterson, H. B. Fischer, E. K. Stodola, H. C. Forbes, A. E. Thiessen, D. E. Foster, H. P. Westman, R. M. Wilmotte

Radio Wave Propagation

C. R. Burrows, *Chairman*

S. L. Bailey, D. H. Menzel, J. L. Barnes, H. R. Mimno, T. J. Carroll, K. A. Norton, J. H. Dellinger, H. O. Peterson, W. A. Fitch, R. L. Smith-Rose, G. D. Gillett, H. P. Thomas, W. C. Lent, W. D. White

Standards

H. A. Wheeler, *Chairman*

Andrew Alford, E. A. Guillemin, R. S. Burnap, R. F. Guy, C. R. Burrows, I. J. Kaar, W. G. Cady, G. G. Muller, C. C. Chambers, C. J. Young, L. F. Curtis, L. E. Whittemore

Symbols

E. W. Schafer, *Chairman*

R. R. Batcher, J. H. Dellinger, M. R. Briggs, E. T. Dickey, R. S. Burnap, H. S. Knowles, C. R. Burrows, O. T. Laube, A. F. Pomeroy

Television

I. J. Kaar, *Chairman*

Hollis Baird, A. G. Jensen, J. E. Brown, L. M. Leeds, A. B. DuMont, H. M. Lewis, D. E. Foster, A. V. Loughren, P. C. Goldmark, A. F. Murray, T. T. Goldsmith, Jr., R. E. Shelby, G. W. Fyler, D. B. Sinclair, D. B. Smith

Radio Transmitters

R. F. Guy, *Chairman*

Andrew Alford, J. F. Morrison, M. R. Briggs, J. C. Shelleng, Harry Diamond, Robert Serrell, F. A. Gunther, D. B. Sinclair, W. E. Jackson, I. R. Weir, W. B. Lodge, J. E. Young

Antennas

Andrew Alford, *Chairman*

G. H. Brown, W. E. Jackson, Harry Diamond, R. W. P. King, W. S. Duttera, W. B. Lodge, Sidney Frankel, J. F. Morrison, R. F. Guy, J. C. Schelleng, D. B. Sinclair

Institute Representatives in Colleges—1944

Alabama Polytechnic Institute: Woodrow Darling
Alberta, University of: J. W. Porteous
Arkansas, University of: P. K. Hudson

British Columbia, University of: H. J. MacLeod
Brooklyn, Polytechnic Institute of: F. E. Canavaciol

California Institute of Technology: S. S. Mackeown
California, University of: L. J. Black
Carleton College: C. A. Culver
Carnegie Institute of Technology: B. R. Teare, Jr.
Case School of Applied Science: P. L. Hoover
Cincinnati, University of: W. C. Osterbrock
Colorado, University of: J. M. Cage
Columbia University: J. B. Russell
Connecticut, University of: P. H. Nelson
Cooper Union: J. B. Sherman
Cornell University: True McLean

Detroit, University of: A. R. Satullo
Drexel Institute of Technology: R. T. Zern
Duke University: W. J. Seeley

Florida, University of: P. H. Craig

Georgia School of Technology: M. A. Honnell

Harvard University: E. L. Chaffee

Idaho, University of: Hubert Hattrup
Illinois Institute of Technology: P. G. Andres
Illinois, University of: A. J. Ebel
Iowa, University of: R. C. Kent

Johns Hopkins University: Ferdinand Hamburger, Jr.

Kansas State College: Karl Martin
Kansas, University of: G. A. Richardson

Lawrence Institute of Technology: H. L. Byerlay
Lehigh University: C. G. Brennecke
Louisiana State University: Taintor Parkinson

Maine, University of: W. J. Creamer, Jr.

Manhattan College: E. N. Lurch
Maryland, University of: G. L. Davies
Massachusetts Institute of Technology: W. H. Radford and E. Guillemin
McGill University: F. S. Howes
Michigan, University of: L. N. Holland
Minnesota, University of: O. A. Becklund

Nebraska, University of: F. W. Norris
Newark College of Engineering: Solomon Fishman
New York, College of the City of: Harold Wolf
New York University: Philip Greenstein
North Dakota, University of: E. J. O'Brien
Northeastern University: G. E. Pihl
Northwestern University: A. B. Bronwell
Notre Dame, University of: H. E. Ellithorn

Ohio State University: E. C. Jordan
Oklahoma Agriculture and Mechanical College: H. T. Fristoe
Oregon State College: A. L. Albert

Pennsylvania State College: G. H. Crossley
Pennsylvania, University of: C. C. Chambers
Pittsburgh, University of: R. C. Gorham
Princeton University: J. G. Barry
Purdue University: R. P. Siskind

Queen's University: H. H. Steward

Rensselaer Polytechnic Institute: H. D. Harris
Rice Institute: C. R. Wischmeyer
Rose Polytechnic Institute: G. R. Schull
Rutgers University: J. L. Potter

Southern Methodist University: R. E. Beam
Stanford University: Karl Spangenberg
Stevens Institute of Technology: F. C. Stockwell

Texas, University of: E. W. Hamlin
Toronto, University of: R. G. Anthes
Tufts College: A. H. Howell

Union College: F. W. Grover
United States Naval Academy: G. R. Giet
Utah, University of: O. C. Haycock

Virginia, University of: L. R. Quarles
Virginia Polytechnic Institute: R. E. Bailey

Washington, University of: A. V. Eastman
Washington University: Stanley Van Wambeck
Wayne University: G. W. Carter
Western Ontario, University of: G. A. Woonton
West Virginia University: R. C. Colwell
Wisconsin, University of: Glenn Koehler
Worcester Polytechnic Institute: H. H. Newell

Yale University: H. M. Turner

Institute Representatives on Other Bodies—1944

American Documentation Institute: J. H. Dellinger
Council of the American Association for the Advancement of Science: J. C. Jensen
Joint Co-ordination Committee on Radio Reception of the E.E.I., N.E.M.A., and R.M.A.: C. E. Brigham
National Research Council, Division of Engineering and Research: F. E. Terman
Radio Technical Planning Board: Haraden Pratt (W. L. Barrow, alternate)
U. R. S. I. (International Scientific Radio Union) Executive Committee: C. M. Jansky, Jr.
U. S. National Committee, Advisers on Electrical Measuring Instruments: Melville Eastham and H. L. Olesen
U. S. National Committee, Advisers on Symbols: L. E. Whittemore and J. W. Horton
U. S. National Committee of the International Electrotechnical Commission: H. M. Turner

ASA Standards Council: Alfred N. Goldsmith (H. P. Westman, alternate)
ASA Board of Examination: H. P. Westman
ASA Electrical Standards Committee: H. M. Turner (H. P. Westman, alternate)
ASA Sectional Committee on Acoustical Measurements and Terminology: E. D. Cook and H. F. Olson
ASA Sectional Committee on Definitions of Electrical Terms: Haraden Pratt
ASA Subcommittee on Vacuum Tubes: B. E. Shackelford
ASA Sectional Committee on Electric and Magnetic Magnitudes and Units: J. H. Dellinger
ASA Sectional Committee on Electrical Installations on Shipboard: I. F. Byrnes
ASA Sectional Committee on Electrical Measuring Instruments: Wilson Aull, Jr.
ASA Sectional Committee on Graphical Symbols and Abbreviations for Use on Drawings: Austin Bailey (H. P. Westman alternate)
ASA Subcommittee on Communication Symbols: H. M. Turner
ASA Sectional Committee on Letter Symbols and Abbreviations for Science and Engineering: H. M. Turner
ASA Subcommittee on Letter Symbols for Radio Use: H. M. Turner
ASA Sectional Committee on National Electrical Safety Code, Subcommittee on Article 810, Radio Broadcast Reception Equipment: E. T. Dickey (Virgil M. Graham, alternate)
ASA Sectional Committee on Preferred Numbers: A. F. Van Dyck
ASA Sectional Committee on Radio: Alfred N. Goldsmith, chairman; Haraden Pratt, and L. E. Whittemore
ASA Sectional Committee on Radio-Electrical Co-ordination: J. V. L. Hogan, C. M. Jansky, Jr., and L. E. Whittemore
ASA Sectional Committee on Specifications for Dry Cells and Batteries: H. M. Turner
ASA Sectional Committee on Standards for Drawings and Drafting Room Practices: Austin Bailey (H. P. Westman, alternate)
ASA Committee on Vacuum Tubes for Industrial Purposes: B. E. Shackelford
ASA War Committee on Radio: Alfred N. Goldsmith*

* Also chairman of its Subcommittee on Insulating Material Specifications for the Military Services.

BOOKS

Books

Shop Job Sheets in Radio, Book II—Service Problems, by Robert Neil Auble

Published by The Macmillan Company, 60 Fifth Avenue, New York, N. Y. 128 pages +ix pages+40 illustrations. 8½×11 inches. Price $1.50.

This is a new book which is intended to give the student, who has made a preliminary study of the fundamental electrical principles of radio, an opportunity to study the application of these principles to standard radio equipment.

Book I, entitled "Shop Job Sheets in Radio" by the same author*, is directed primarily to the beginner in radio, to assist him in obtaining a practical working knowledge of the electrical fundamentals of radio, including such fundamental components as transformers, capacitors, resistors, and some of the elementary laws of electricity, such as Ohm's law and Lenz's law. This text is intended for use following Book I by students wishing some knowledge of installation, maintenance, etc. This book should serve as a good laboratory manual in the training of beginnners for installation, maintenance, and assembly work. It consists essentially of some 25 job sheets; each of which covers several pages. The job sheets, each intended as one lesson or exercise, are divided into the following sections:

- Objectives
- References
- Topics for classroom discussion
  - Related knowledge
  - Related skills
- What to do
- Optional exercises
- Question section

Each question in the last section has ample space provided for an answer including a diagram when required.

This text is intensely practical and approaches its subject from the standpoint that all radio equipments are constructed from five basic component parts: resistors, condensers, transformers, inductors, and vacuum tubes. This book covers the testing of resistors, condensers, transformers, filter chokes, vacuum tubes, the construction of power supply, audio amplifiers, drivers, power amplifiers, etc. About one third of the book is devoted to a discussion of the principles, the construction, and the testing of the superheterodyne receiver and its fundamental circuit components. The latter third of the book is devoted to a similar study of transmitters including such items as the Hartley oscillator, the electron-coupled oscillator, the quartz-crystal oscillator, radio-frequency amplifiers, frequency-doubler circuits, modulation and transmitter power supplies. Formulas are omitted almost completely throughout the text, and the treatment of each problem is most elementary. Reference reading as well as detailed explanation by the instructor is required. Some 12 reference books and manuals are included in the bibliography, among them being tube manuals and texts for the radio serviceman.

The order in which the material is presented is logical, the arrangement is good, and the exercises are well prepared. The illustrations appear to be ample and may easily be interpreted without the aid of an instructor. The author provides ample space for notes. Since all of the student work sheets are bound in the book together with the instructor's corrections and grading, this text can be used by the student for reference after completion of the course. The book appears to be free of obvious errors and covers the intended subject matter adequately. This is the best text of its type that has come to this reviewer's attention.

F. X. Rettenmeyer
Radio Corporation of America
Camden, New Jersey

* Proc. I.R.E., vol. 32, p. 575; September, 1944.

Modern Operational Mathematics in Engineering, by Ruel V. Churchill

Published (1944) by McGraw-Hill Book Company, 330 W. 42 St., New York 18, N. Y. 302 pages+4-page index+vi pages. 105 figures. 5×8 inches. Price, $3.50.

This volume is offered as a companion to the same author's "Fourier Series and Boundary Value Problems" (1941)* dealing with the problems of harmonic vibrations or bounded space. A reader studying both subjects would benefit by the author's similar technique and terminology to be expected in both books.

The subject is mainly the transient phenomena of electric currents, mechanical displacements, and heat transfer, all of major interest to the radio engineer. Since the author is primarily a mathematician, the treatment is from that point of view but he has attempted to minimize the attention to the rigors of pure mathematics which irk the engineer.

Half of the space (chapters I to IV and X) is directed to the more elementary treatment of Laplace transforms and their application without reference to complex variables. This part is intended for undergraduate students. It gives a mathematical introduction with a number of elementary examples well chosen for their variety of subject matter and range of difficulty. Unfortunately, there is no descriptive introduction to the nature and purpose and uses of these processes, so that must be supplied to the student by his instructor or to the reader from his previous knowledge. In fact, the entire volume suffers from the lack of word descriptions covering the significance of the processes and the utility of the examples.

The other half of the space includes a brief introduction to the functions of a complex variable, followed by the more advanced cases of Laplace transforms (Fourier integrals). It is intended for graduate students and is recommended to follow a more elementary study of the subject. Here also, the examples are commendable for their variety and utility.

It is presumptive that the author composed this volume to fill a need not met by prior texts, perhaps a need for a compact treatment without the encumbrance of too much mathematical background. His treatment is instructive to one who already appreciates the significance and technique of this method. It is arranged to be convenient as a reference and includes several short tables of Laplace transforms. The terminology differs from that of other tables in use.

H. A. Wheeler
Hazeltine Corporation
Little Neck, L. I., N. Y.

* Proc. I.R.E., vol. 29, p. 655, December, 1941.

Radio Waves and the Ionosphere, by T. W. Bennington

Published (1944) by *Wireless World*, Iliffe and Sons, Ltd., Dorset House, Stamford Street, London, S. E. 1., England. 81 pages+vi pages. 27 figures. 5×7½ inches. Price, 6/-net.

In the last twenty years much work has been done and many papers written on the propagation of radio waves in the ionosphere, and the mechanism whereby long-distance radio propagation is possible. Most of the literature on the subject is highly technical, and scattered throughout various journals. There has thus been a need for an abbreviated popular monograph on the subject, capable of making radio technicians and operators "ionosphere-minded," and assisting them to get a basic picture of what their radio waves do between the transmitting antenna and the receiving antenna.

The purpose of this book is to fulfill the above need. It is always difficult to write in a popular manner of a subject as complex as this, which involves, for clarity and consistency of understanding, a familiarity with radiation, atomic structure, astrophysics, and Maxwell's equations. Consequently there are places in the book where inadequate and even inconsistent explanations are given, although the over-all picture is a good one. For example, no good picture of low-frequency propagation is given, the distribution of ionization in the atmosphere is not quite correct, and some highly speculative material is presented, which the casual reader might accept as established fact. Also the rather involved, though fundamental, subject of group and phase velocity is avoided, with some consequent loss of clarity.

On the whole the book is well written and easy to read and summarizes at least the fundamentals and early work on the ionosphere. It will undoubtedly be of interest and use to the radio technician who may have hitherto been unaware of the importance of the role of the ionosphere in radio propagation.

Newbern Smith
National Bureau of Standards
Washington, D. C.

Contributors

Alexander B. Bereskin

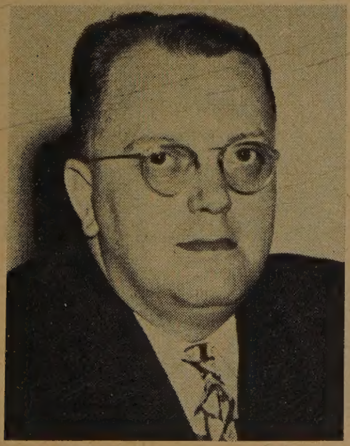
Alexander B. Bereskin (A'41) was born in San Francisco, California, on November 15, 1912. He received the E.E. degree from the University of Cincinnati in 1935. This was followed by work with the Commonwealth Manufacturing Corporation from 1935 to 1937 and with the Cincinnati Gas and Electric Company from 1937 to 1939. In 1939 he returned to the University of Cincinnati as a teaching Fellow and received the M.Sc. degree in engineering in 1941. From 1941 to 1943 Mr. Bereskin was connected with the electrical engineering department of the University of Cincinnati as an instructor, and as assistant professor since that time. He is a member of Sigma Xi and the A.I.E.E.

❖

En-lung Chu is a graduate of Chiao-Tung University in China. After graduation, he served for a year as an engineer in the power plant of the municipality of Hangchow, Chekiang. He then went to the Institute of Physics of the Academia Sinica in Kweilin to do research on radio problems.

❖

R. H. DeLany joined the engineering department of the United Broadcasting Company in July, 1929; was assistant to the technical supervisor in the building of the transmitter plant for station WHK; in 1931, he assisted in the building of new studios for WHK; from then until early 1942 he was in charge of studio plant wiring and maintenance and remote-control facilities. Mr. DeLany was made assistant chief engineer in 1942 and chief engineer of WHK in 1944.



R. H. DeLany

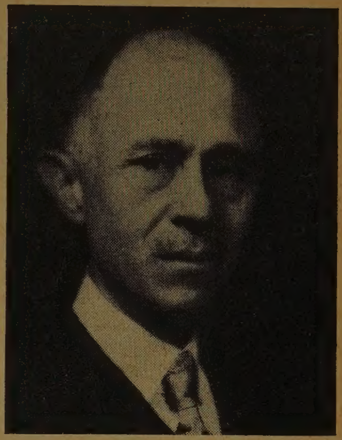
Donald Foster was born on March 19, 1900, at West Torbrook, Nova Scotia. He was educated at Acadia University and at Yale University, where he received a Ph.D. degree in physics in 1924. He is at present at Harvard University, on leave of absence from Stevens Institute of Technology, where he taught physics and mathematics. Before becoming a teacher, Professor Foster spent a year designing optical apparatus at the Eastman Kodak Company, a year in vacuum-tube engineering at the Westinghouse Electric and Manufacturing Company in Bloomfield, N. J., and eleven years at Bel- Telephone Laboratories, where he did experimental research in ferromagnetism and acoustics. At Bell Laboratories he also did some work in applied mathematics, including the theory of the rhombic antenna.

Donald Foster

❖

Frederick Warren Grover (M'17-SM'43) was born in Lynn, Massachusetts, on September 3, 1876. He received the B.S. degree from the Massachusetts Institute of Technology in 1899, the M.S. degree from Wesleyan University in 1901, the Ph.D. degree from George Washington University in 1907, and the Ph.D. degree of the Ludwig-Maximilian University, Munich, in 1908.

He served as assistant in physics and astronomy at Wesleyan University from 1899 to 1901, and as instructor of electrical engineering at Lafayette College during 1901 and 1902. From 1902 to 1907, and from 1908 to 1911, Dr. Grover was, successively, laboratory assistant, assistant physicist, and associate physicist at the National Bureau of Standards. He was head of the department of physics at Colby College from 1911 to 1920, and since that time has been at Union College, first as associate professor, and, since 1932, as professor of electrical engineering. From 1918 to 1933, he was connected with the Bureau of Standards as consulting physicist. He was one of the authors of the Signal Corps textbook, "Principles Underlying Radio Communication," published in 1918. He is a member of Sigma Xi, Eta Kappa Nu, and the American Institute of Electrical Engineers, and a Fellow of the American Physical Society and the American Association for the Advancement of Science.

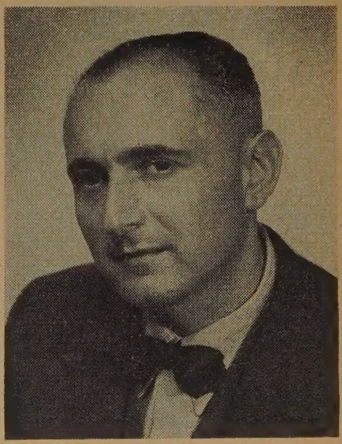


Frederick W. Grover

❖

Henry P. Kalmus (A '39) was born in 1906 in Vienna. He was graduated from the Technical University at Vienna in 1930.

From 1930 to 1938 he was with the Orion Radio Corporation (Subsidiary of Tungsram) in Budapest, Hungary, as head of their laboratories. During this period he designed receivers for Sweden, Belgium, Switzerland and other European countries. From 1939 to 1941 Mr. Kalmus was development engineer for Emerson Radio Corporation in New York. Since that time he has been with the Zenith Radio Corporation in Chicago as research engineer. His investigations include the fields of ultra-high frequency, frequency modulation, and television.



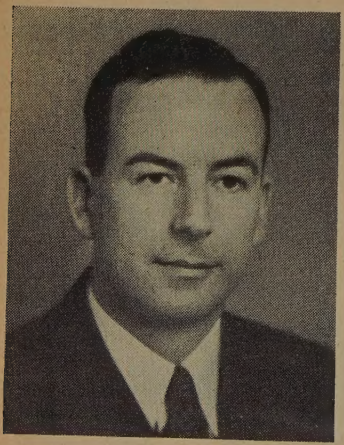
Henry P. Kalmus

R. B. Kerr

Robert B. Kerr was born on November 1, 1912, at Ilion, New York. In 1935 he was graduated from Rensselaer Polytechnic Institute with a degree in electrical engineering, and spent the following two years in the radio engineering department of the General Electric Company. He then joined the geophysical department of the Magnolia Petroleum Company where he was employed for five years. Mr. Kerr is now on leave to the United States Navy in which he holds a Lieutenant (jg) Commission and is on duty in the European theatre of operations.

❖

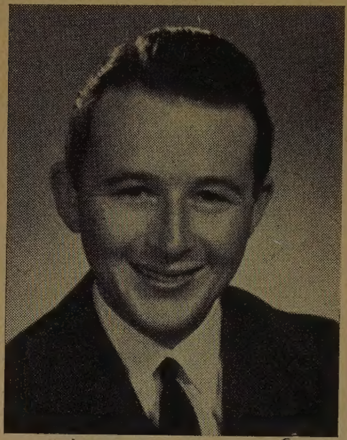
H. W. Kohler (A '34) was born in Thun, Switzerland. He was graduated in electrical engineering from the Federal Polytechnic Institute in Zurich, Switzerland, in 1925. During 1926–1927 he was with the Laboratory of Scintilla Magneto Corporation at Soleure, Switzerland. From 1927 to 1932 he was with the Department of Development and Research, American Telephone and Telegraph Company, working with the Inductive Co-ordination Group on interference problems arising from railroad electrifications. In 1936 he received the Sc.D. degree from Cruft Laboratory, Harvard University, and he was on the instructing staff during 1935–1936. Dr. Kohler was in the Transmitter Laboratory of the Radiomarine Corporation of America in 1937; the Naval Research Laboratory, from 1937 to 1942, doing theoretical and laboratory work on superfrequencies; and the Radio Development Section of the Civil Aeronautics Administration, from 1942 to date, engaged in theoretical studies and field work on very-high-frequency radio ranges and localizers and on interference problems.



H. W. Kohler

❖

John W. Miles was born on December 1, 1920, in Cincinnati, Ohio. He received the B.S. degree in electrical engineering in 1942, the M.S. degree in electrical engineering and the M.S. degree in aeronautical engineering, all from the California Institute of Technology.



John W. Miles

In the summer of 1942 Mr. Miles was associated with the General Electric research laboratory, and is now a teaching fellow at California Institute of Technology in Pasadena, California.

Mr. Miles has held amateur radio license W6LEA since 1935. He is a member of the American Institute of Electrical Engineers, Tau Beta Pi, and Sigma Xi.

❖

R. W. Olson

Robert W. Olson was born on February 17, 1915, at Preston, Minnesota. In 1938 he received a B.E.E. degree from the University of Minnesota, under the Communications option. He was employed by the Magnolia Petroleum Company in Dallas, Texas, as a radio engineer in their geological laboratory from 1938 to 1940. In 1940, Mr. Olson was granted a leave of absence to accept a position with the Navy Department for the duration, and was employed by the Naval Ordnance Laboratory in Washington, D.C., until 1942, when he was transferred to the engineering division of the Bureau of Aeronautics for work on airborne electronic equipment. He has held this post to date.

❖

Earley M. Shook (A '43) was born on December 21, 1901, at Rusk, Texas. He received the B.S. degree in electrical engineering from Texas Agricultural and Mechanical College, in 1925. He worked on test at the General Electric Company, Schenectady, N.Y., in 1925 and 1926. He was assistant division engineer with the Texas Power and Light Company from 1926 to 1935 when he became associated with the electrical department of the Texas Centennial Exposition during its construction and operation.

From 1936 to 1942 Mr. Shook was laboratory engineer in the seismograph department of the Magnolia Petroleum Company at Dallas, Texas, where he was concerned chiefly with the design and development of exploration seismograph instruments and the application of radio communication to exploration operations.

Radio has been his hobby for many years. He has been active in all phases of amateur radio and has held the call W5IT since 1930.

Mr. Shook joined the Navy in May, 1942. He is now Lieutenant Commander, and is stationed in Washington, in the office of the Vice-Chief of Naval Operations.

E. M. Shook

❖

For a biographical sketch of W. L. Everitt see the Proceedings for September, 1944, and frontispiece, this issue.